



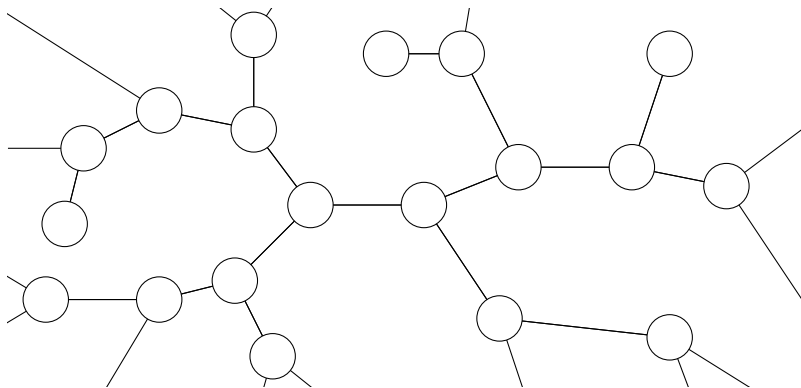
Aalto University
School of Science

Locally Checkable Labeling Problems in Rooted Trees in the Online-LOCAL Model of Computation

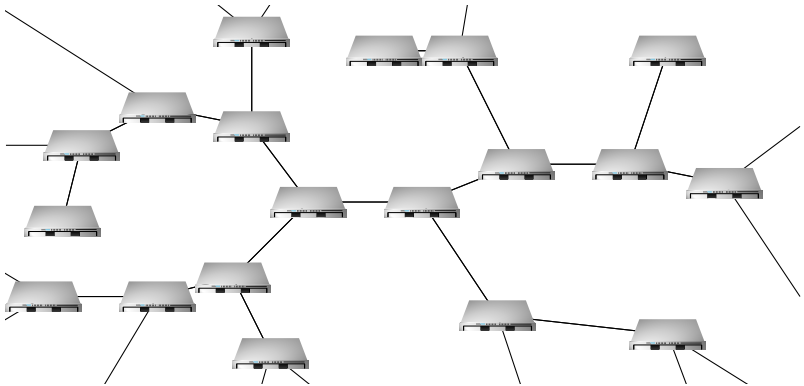
Henrik Lievonen

21.2.2022

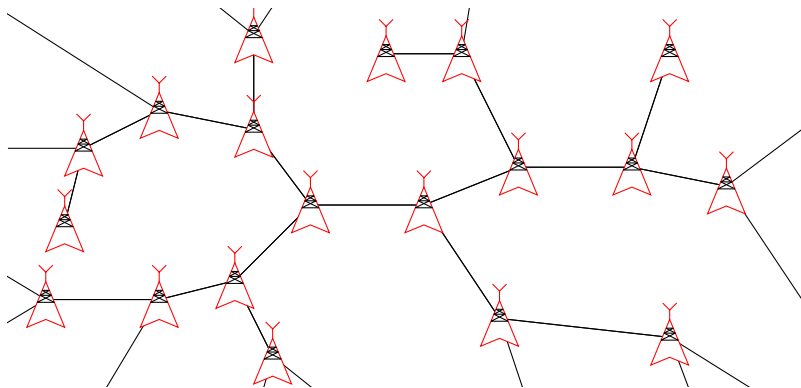
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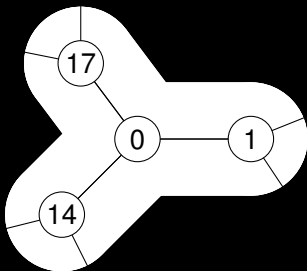
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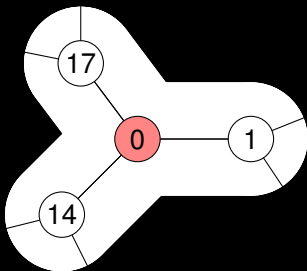
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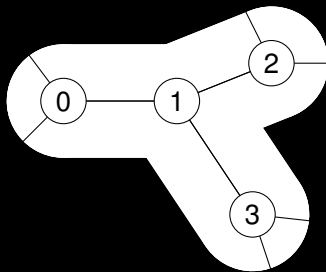
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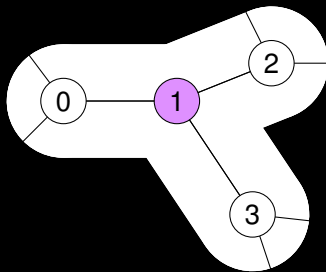
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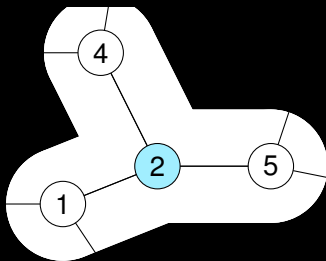
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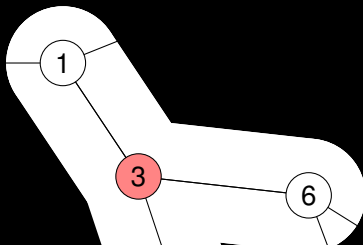
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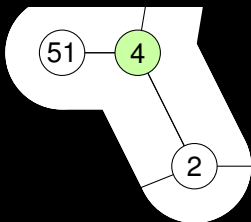
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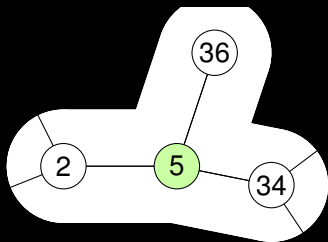
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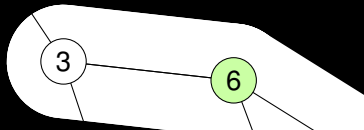
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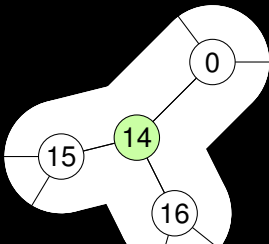
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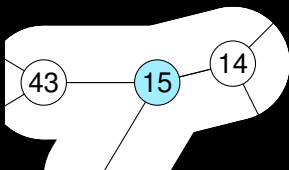
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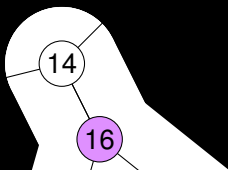
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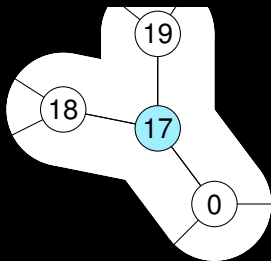
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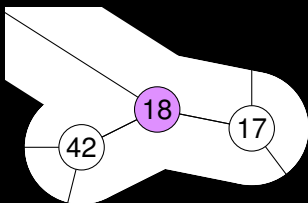
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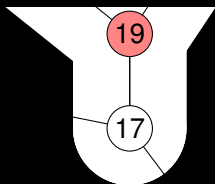
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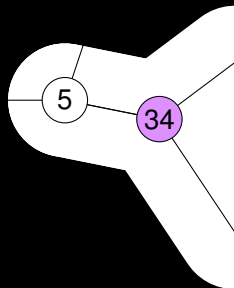
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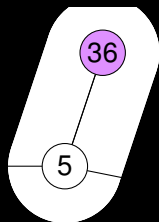
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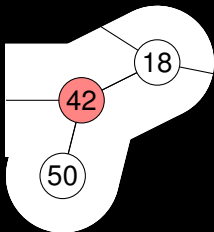
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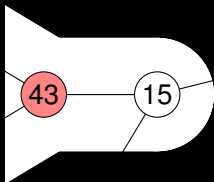
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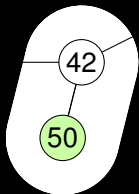
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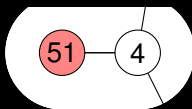
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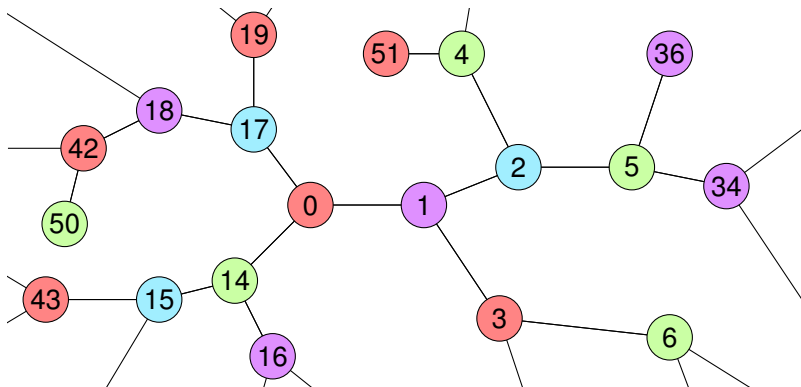
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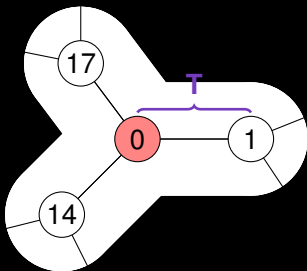
The LOCAL model



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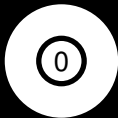
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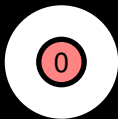
Online graph algorithms



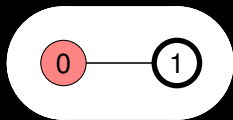
Online graph algorithms



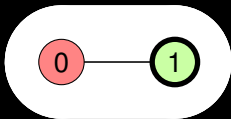
Online graph algorithms



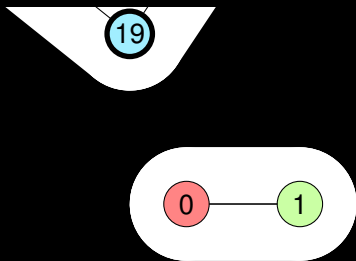
Online graph algorithms



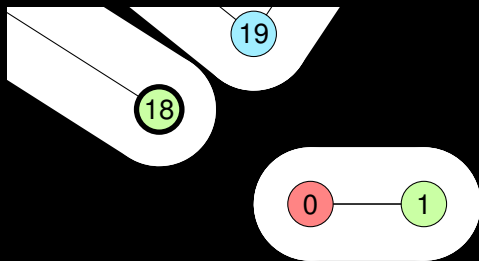
Online graph algorithms



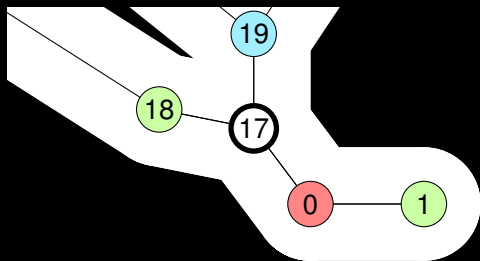
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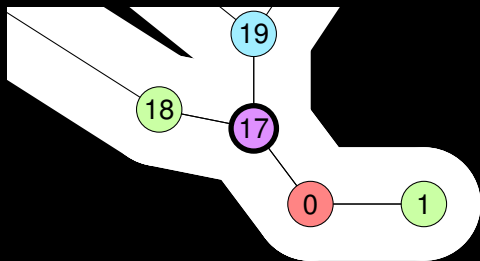
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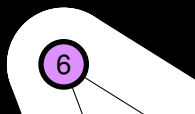
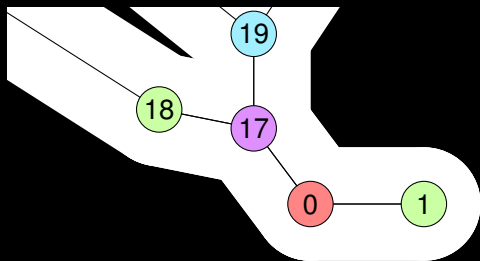
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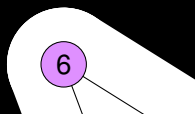
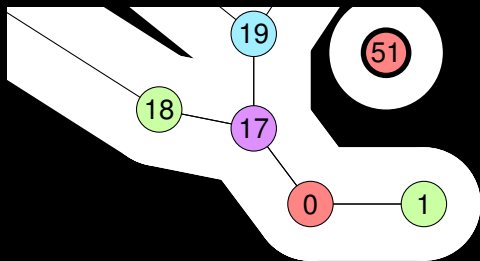
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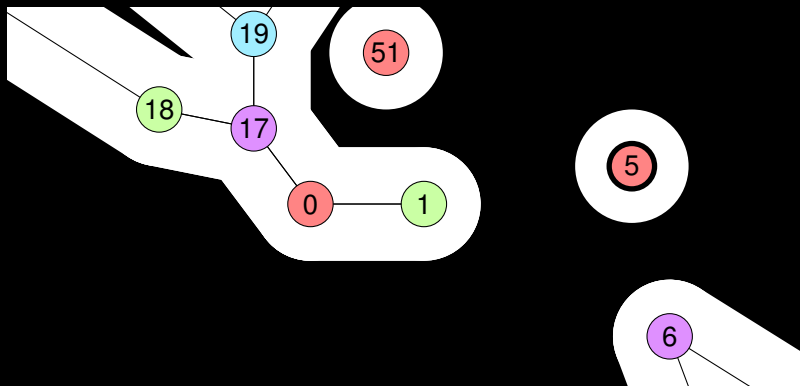
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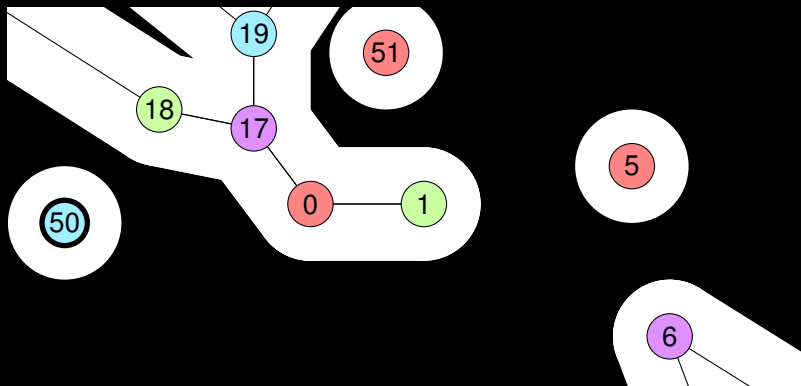
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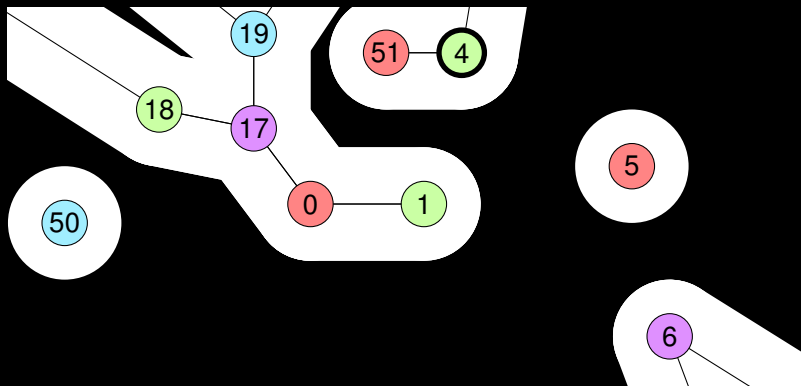
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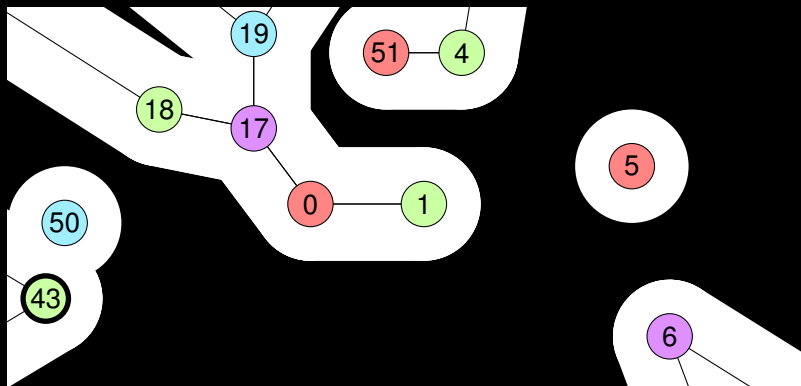
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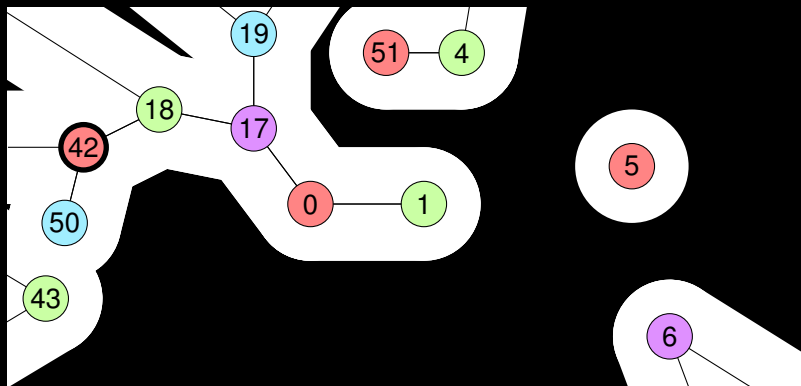
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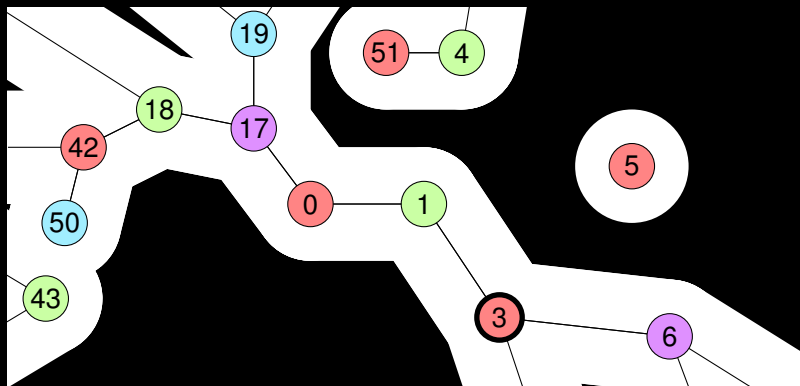
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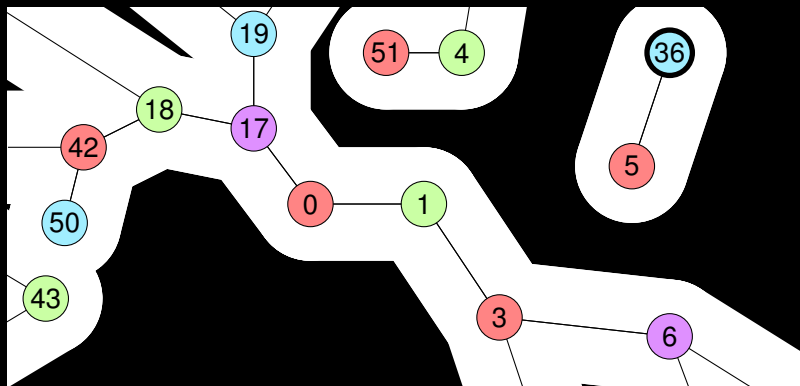
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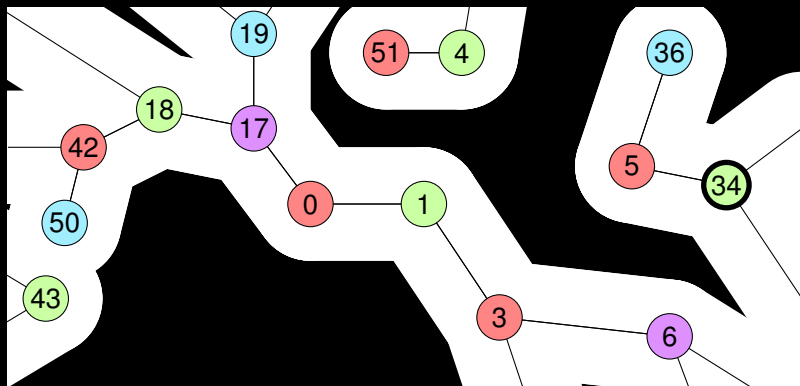
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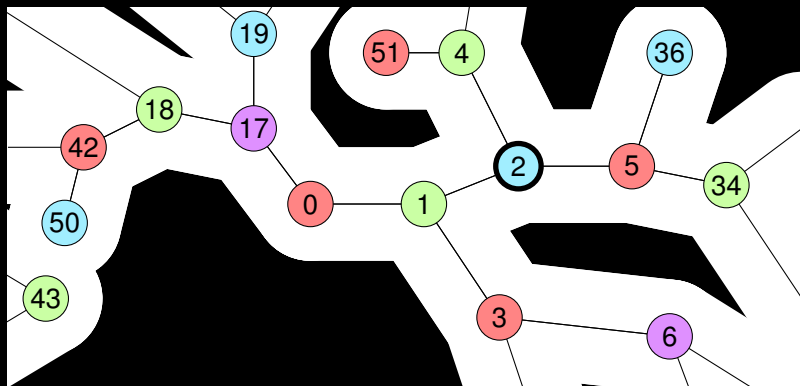
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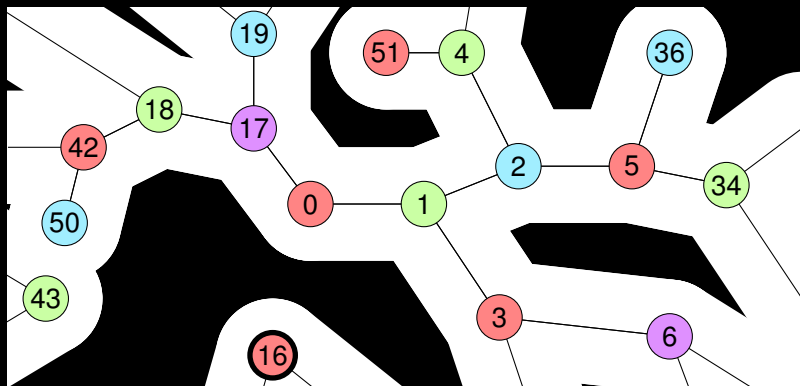
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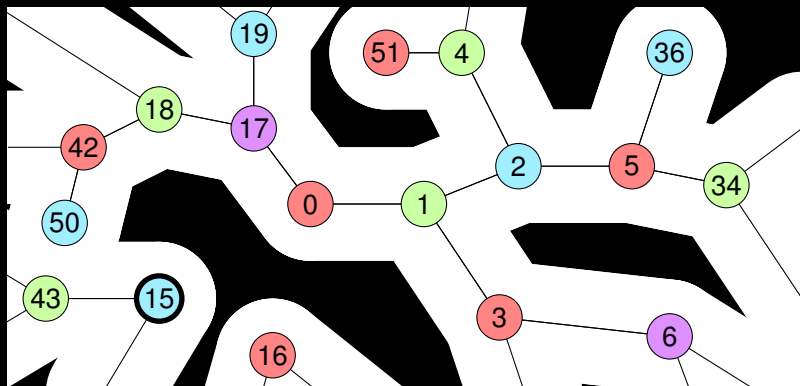
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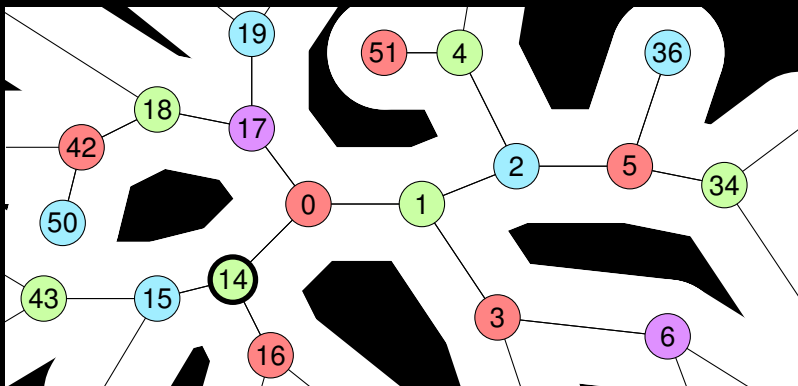
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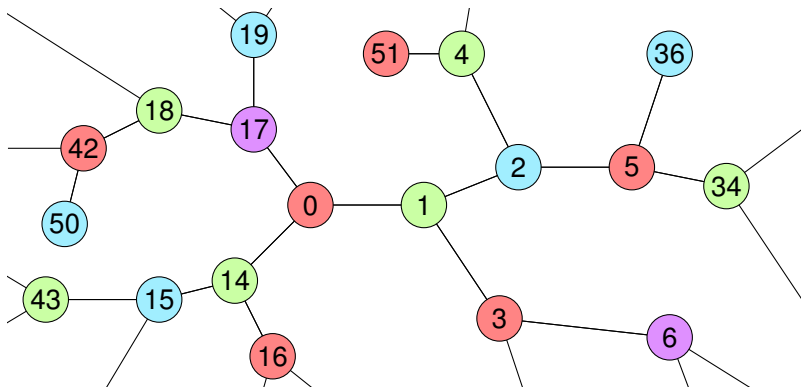
Online graph algorithms



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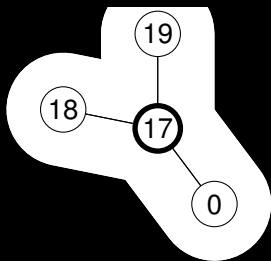
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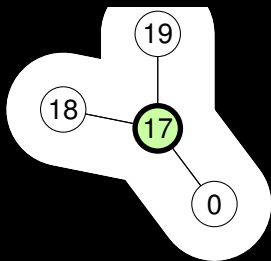
The online-LOCAL model



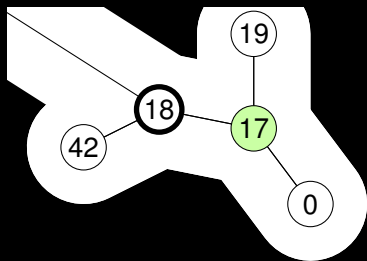
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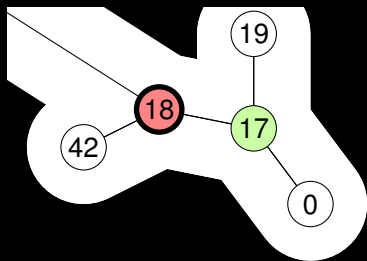
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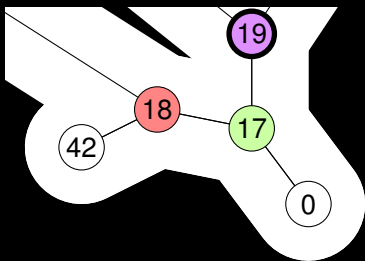
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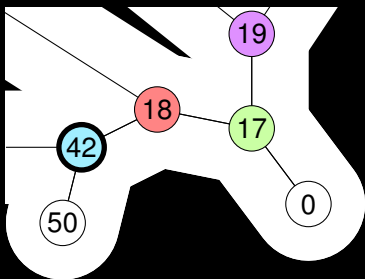
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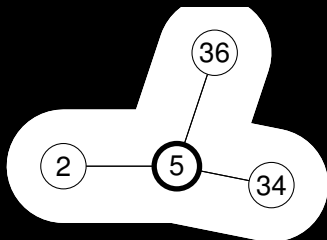
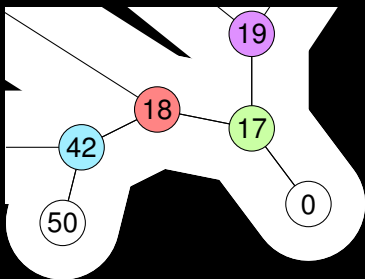
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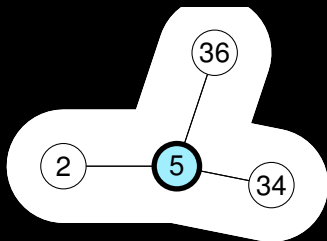
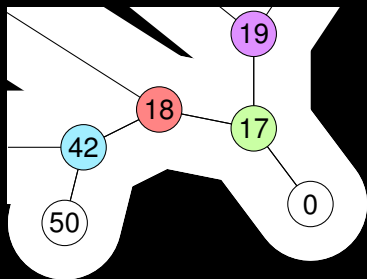
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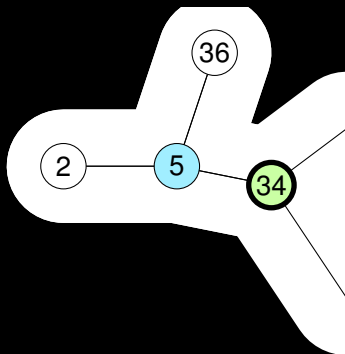
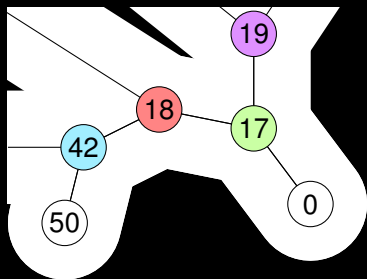
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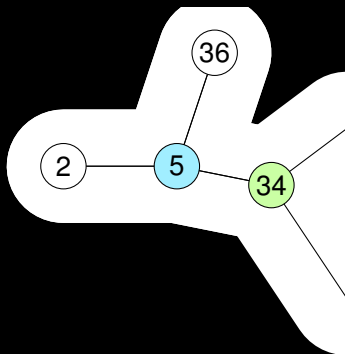
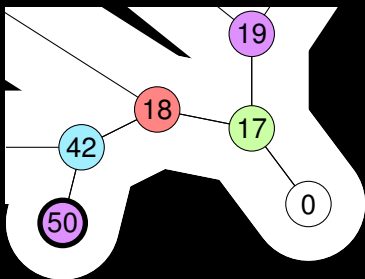
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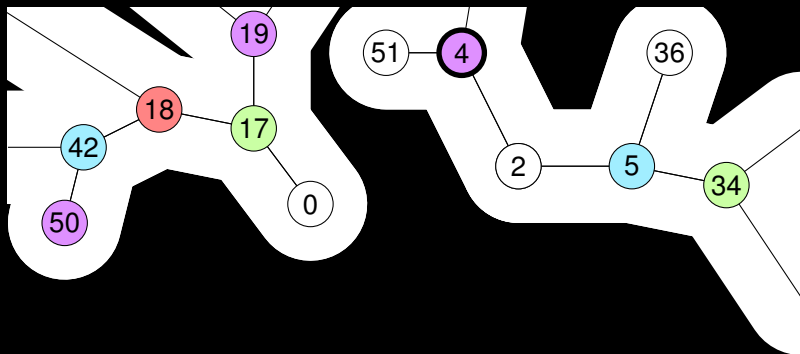
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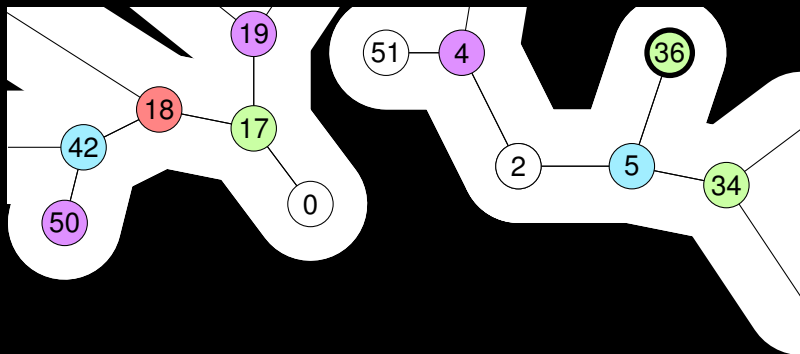
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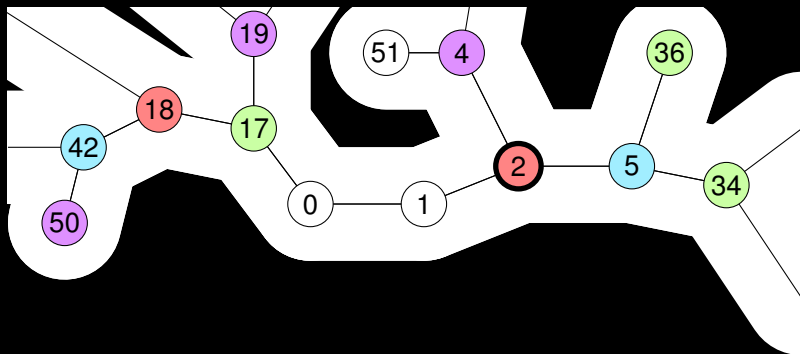
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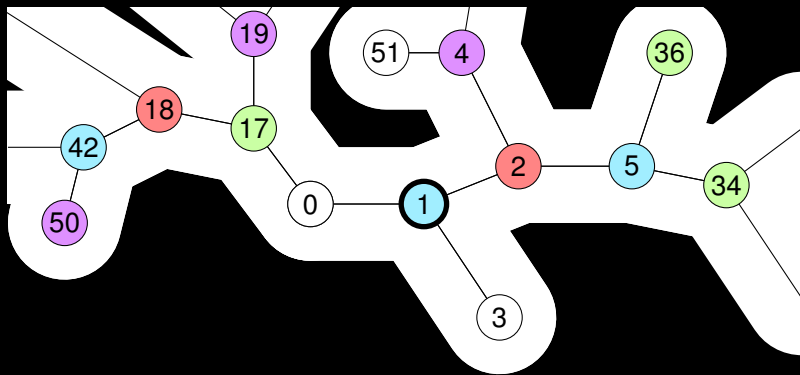
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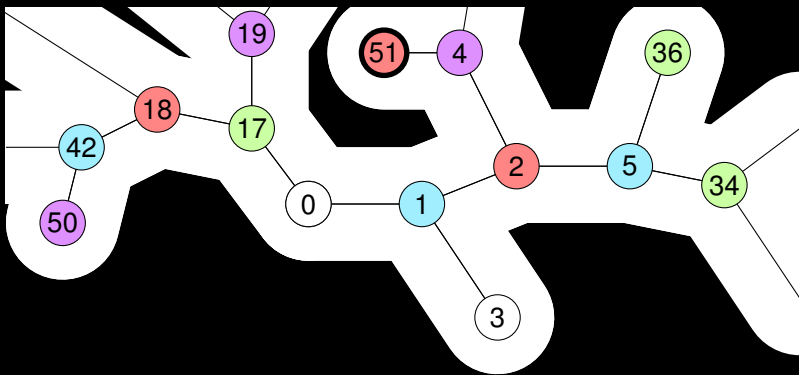
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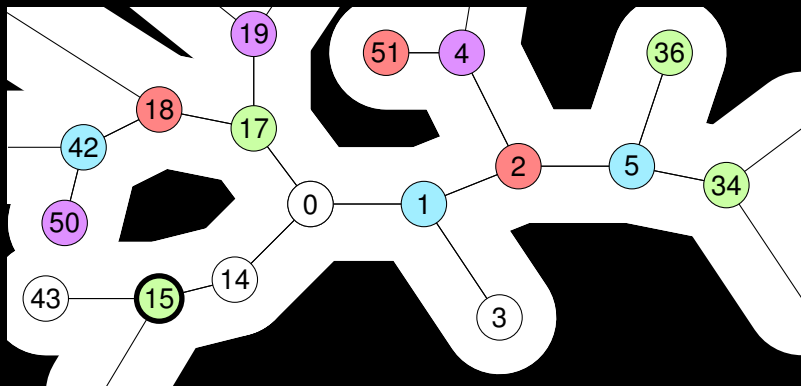
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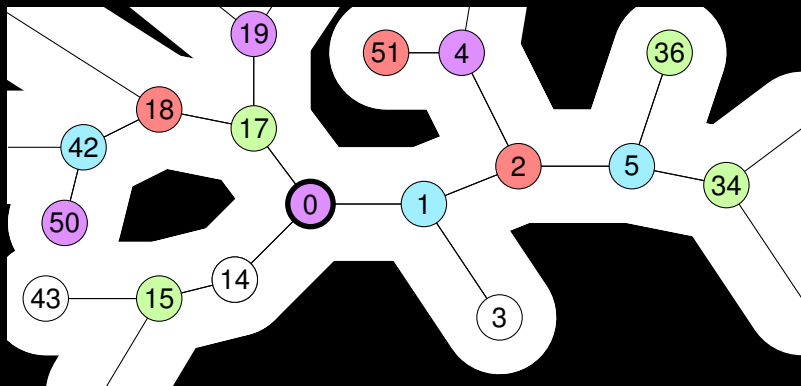
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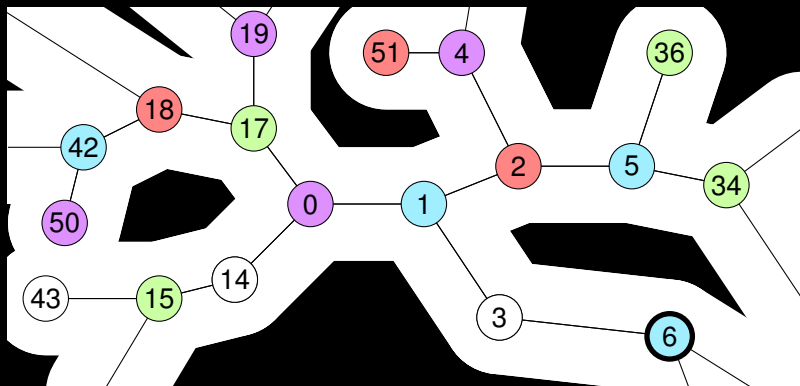
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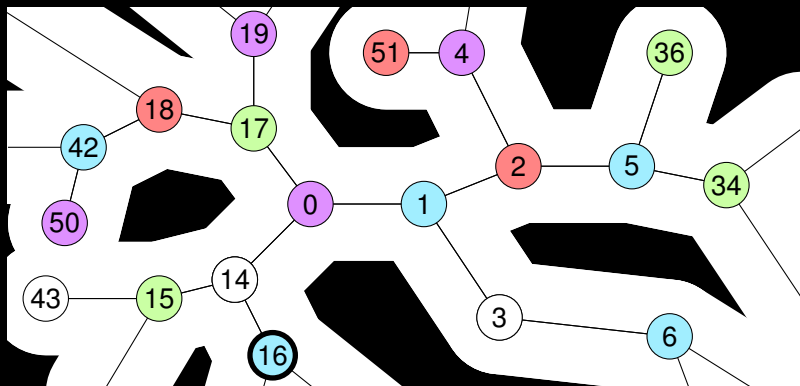
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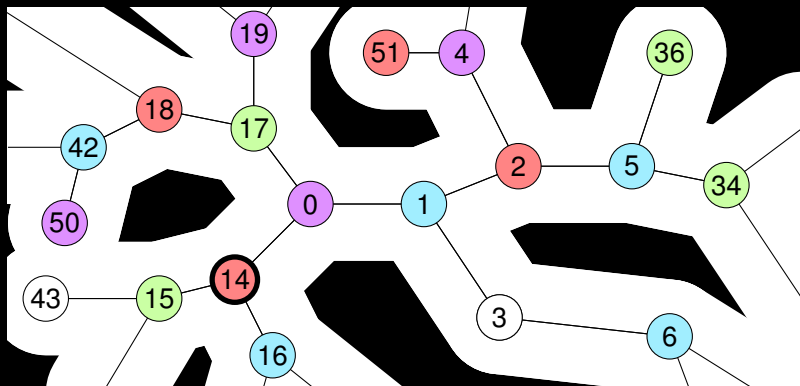
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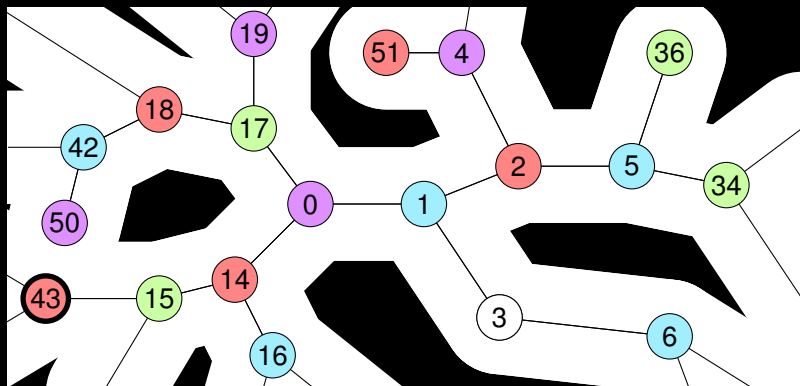
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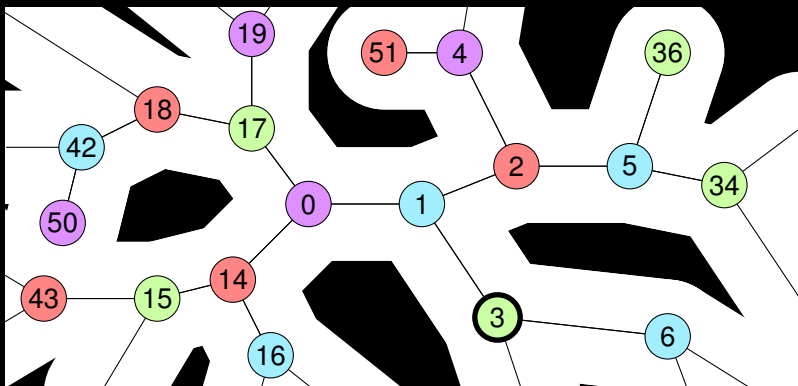
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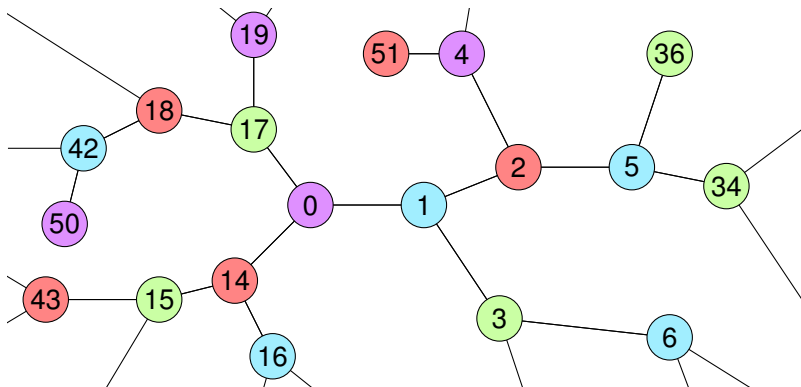
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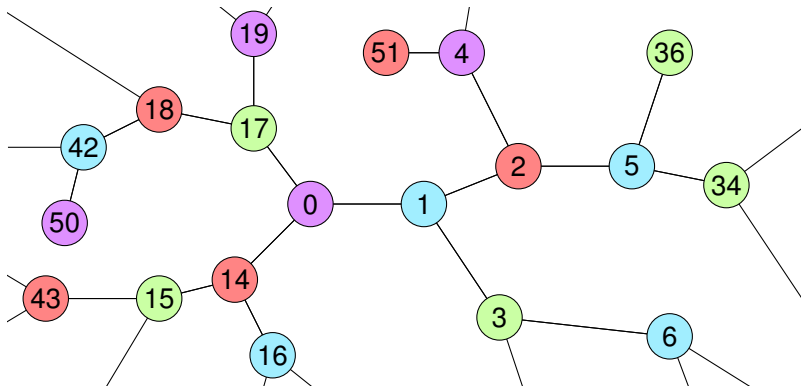
Main contributions

	LOCAL		online- LOCAL
LCLs in directed paths	$O(\log^* n)$	$\Leftrightarrow$	$O(1)$
	$\Theta(n)$	$\Leftrightarrow$	$\Theta(n)$
LCLs in rooted regular trees	$O(\log^* n)$	$\Leftrightarrow$	$O(1)$
	$\Theta(\log n)$	$\Leftrightarrow$	$\Theta(\log n)$
	$n^{\Theta(1)}$	$\Leftrightarrow$	$n^{\Theta(1)}$

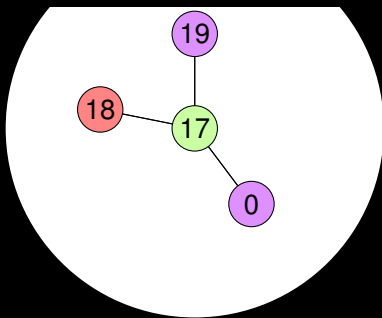
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LCLs in directed paths	$O(\log^* n)$	$\Leftrightarrow$	$O(1)$
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	$\Theta(\log n)$	$\Leftrightarrow$	$\Theta(\log n)$
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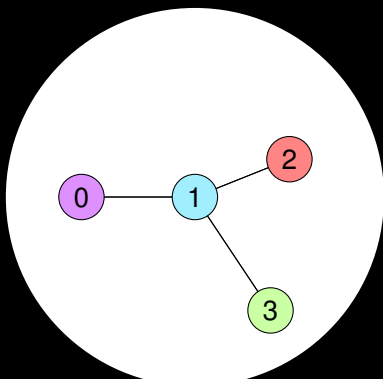
LCL problems



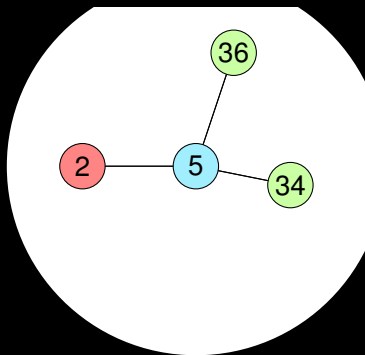
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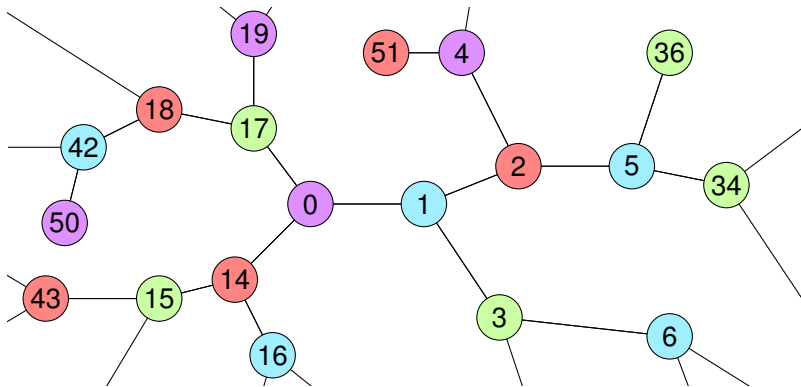
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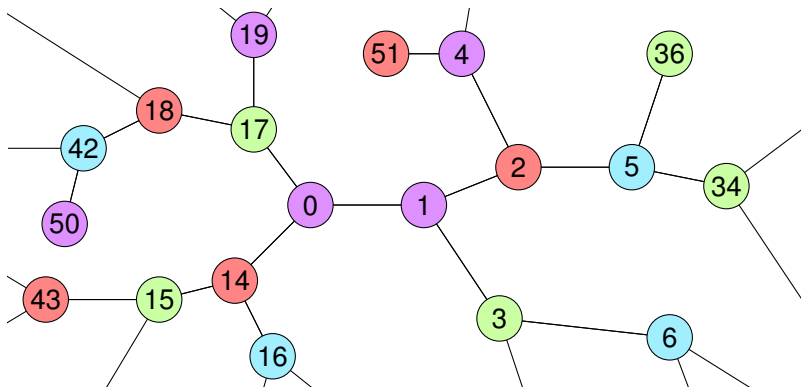
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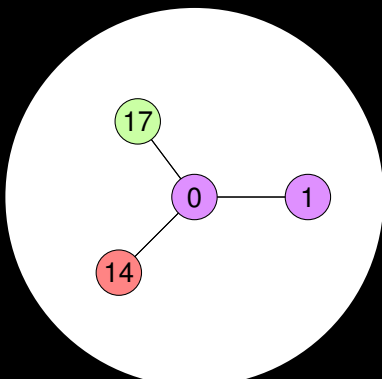
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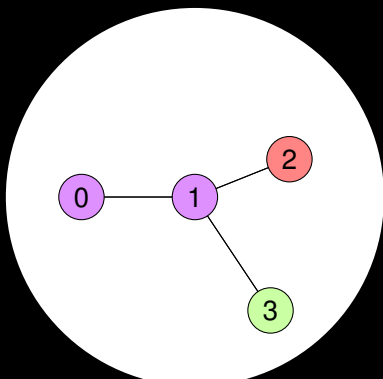
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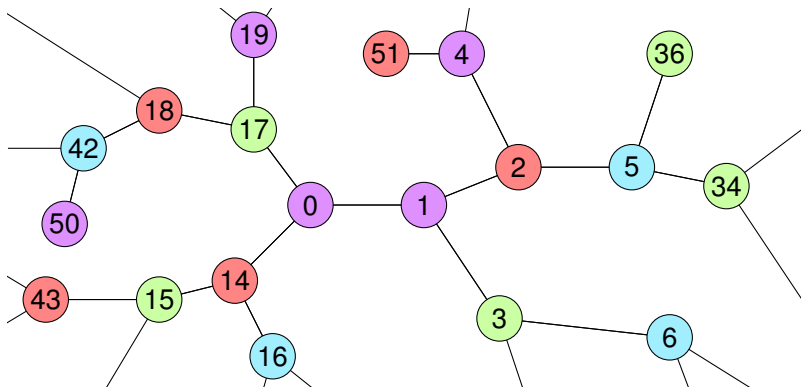
LCL problems



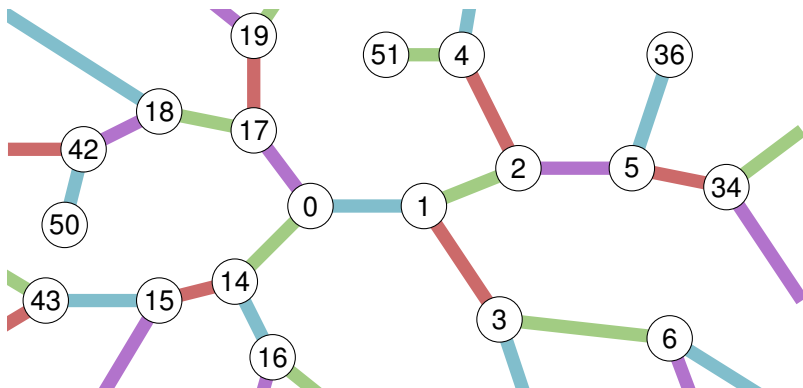
LCL problems



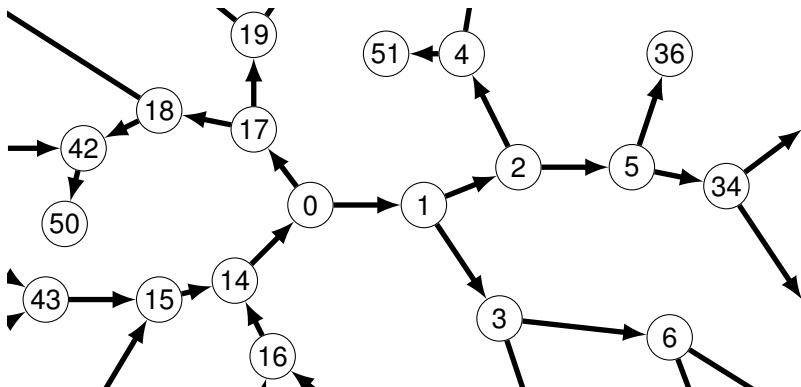
LCL problems



LCL problems — Edge coloring



LCL problems — Sinkless orientation

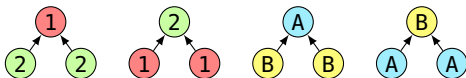


Example — $2\frac{1}{2}$ -coloring problem

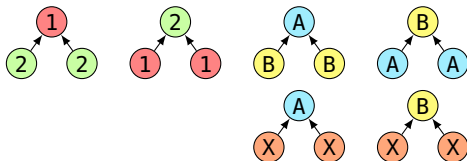
Example — $2\frac{1}{2}$ -coloring problem



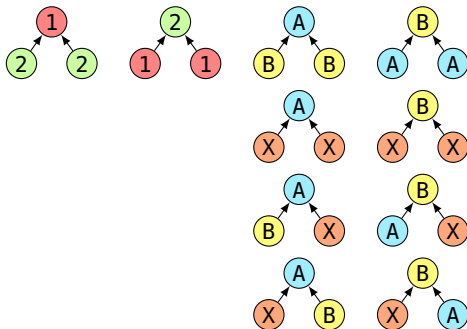
Example — $2\frac{1}{2}$ -coloring problem



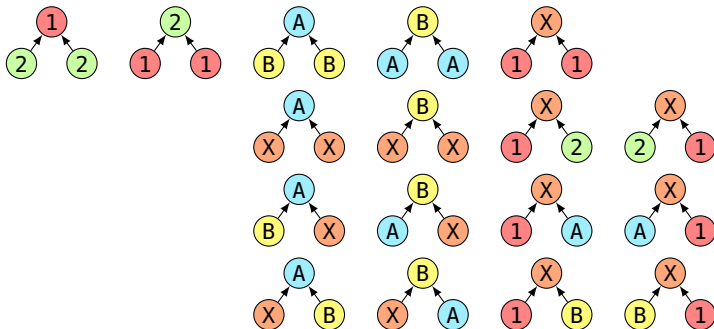
Example — $2\frac{1}{2}$ -coloring problem



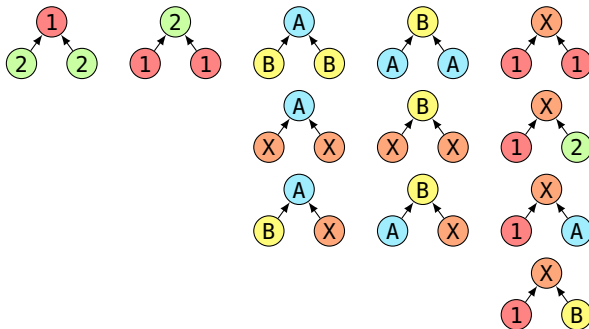
Example — $2\frac{1}{2}$ -coloring problem



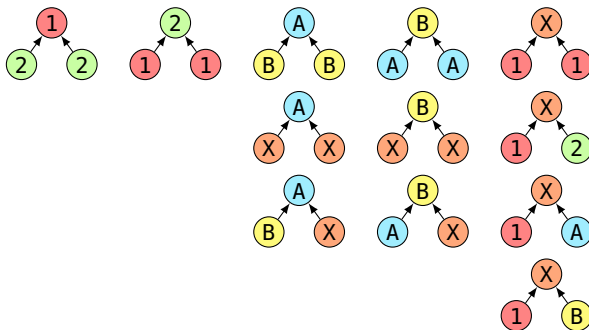
Example — $2\frac{1}{2}$ -coloring problem



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Claim: $2\frac{1}{2}$ -coloring requires $\Omega(\sqrt{n})$ in online-LOCAL.

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in online-LOCAL.

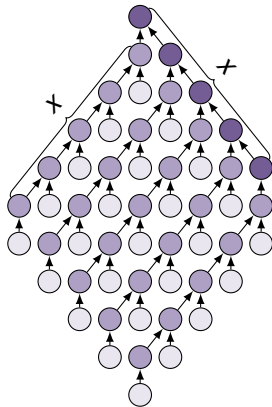
Example — $2\frac{1}{2}$ -coloring problem

Claim:

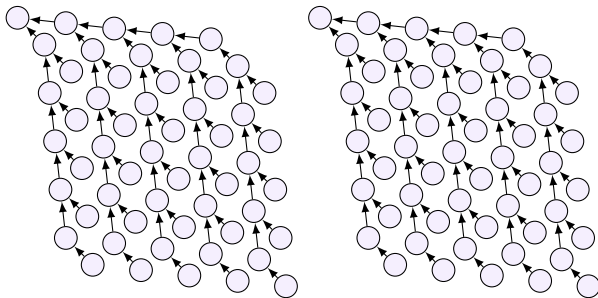
$2\frac{1}{2}$ -coloring
requires $\Omega(\sqrt{n})$
in online-LOCAL.

Assumption:

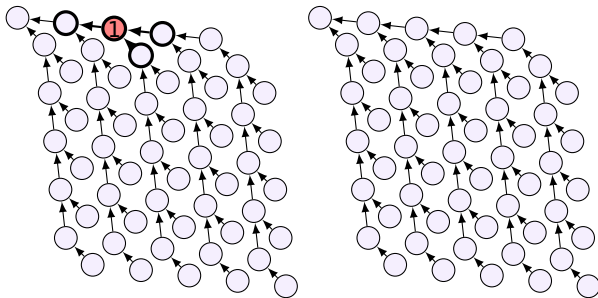
$2\frac{1}{2}$ -coloring
is solvable in $o(\sqrt{n})$
in online-LOCAL.



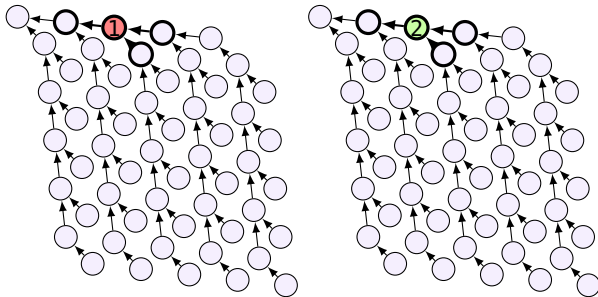
Example — $2\frac{1}{2}$ -coloring problem



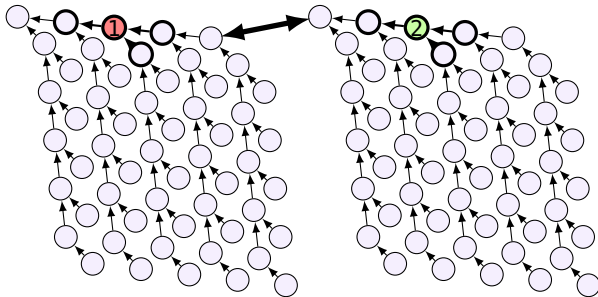
Example — $2\frac{1}{2}$ -coloring problem



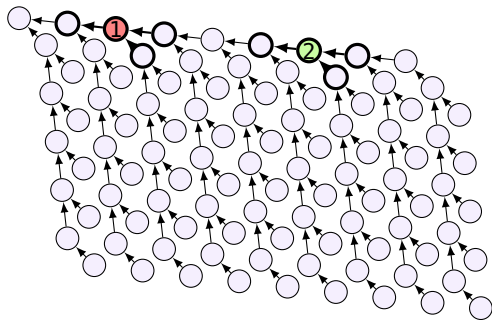
Example — $2\frac{1}{2}$ -coloring problem



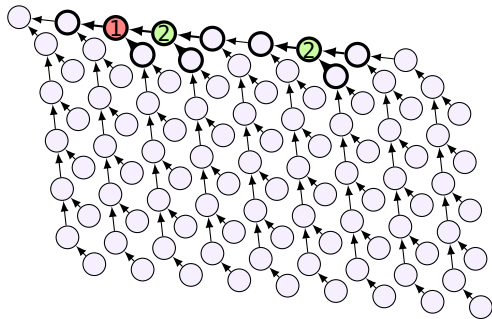
Example — $2\frac{1}{2}$ -coloring problem



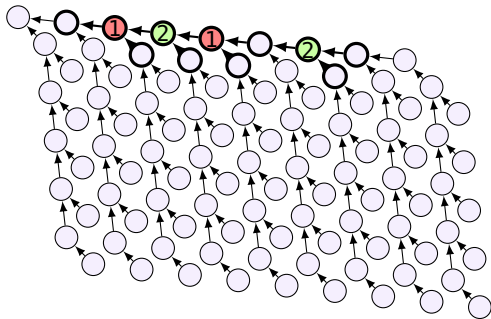
Example — $2\frac{1}{2}$ -coloring problem



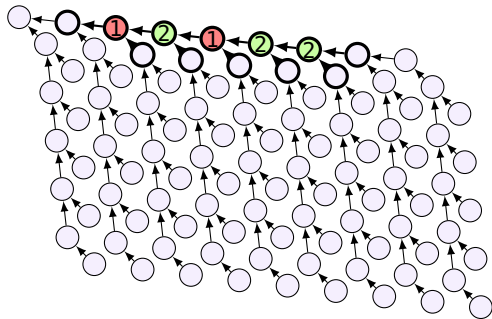
Example — $2\frac{1}{2}$ -coloring problem



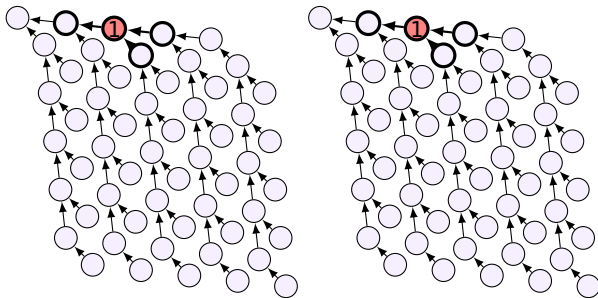
Example — $2\frac{1}{2}$ -coloring problem



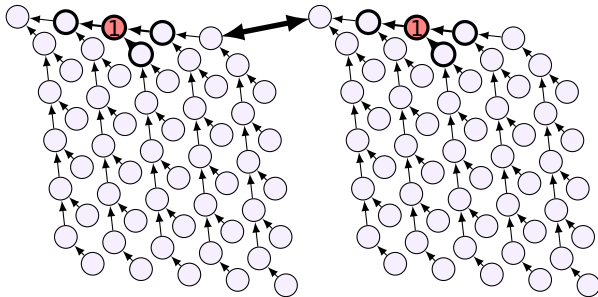
Example — $2\frac{1}{2}$ -coloring problem



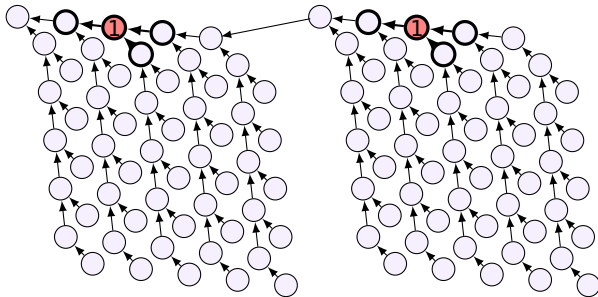
Example — $2\frac{1}{2}$ -coloring problem



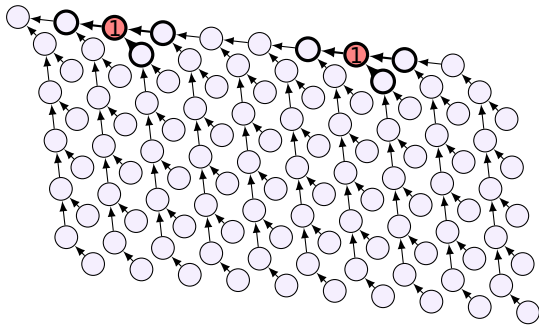
Example — $2\frac{1}{2}$ -coloring problem



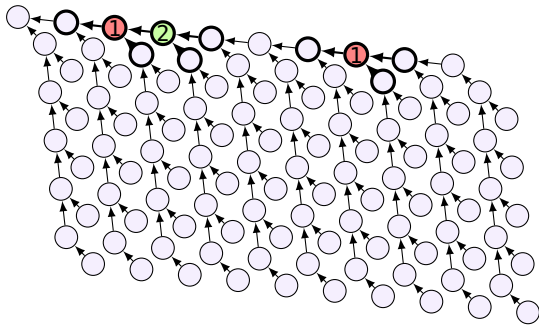
Example — $2\frac{1}{2}$ -coloring problem



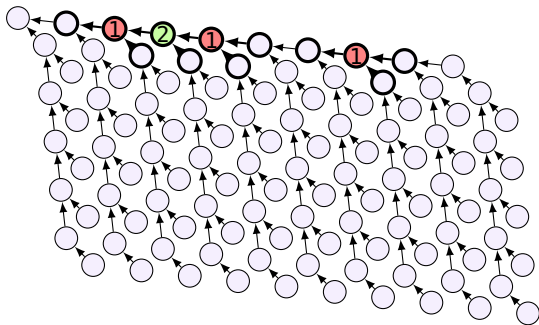
Example — $2\frac{1}{2}$ -coloring problem



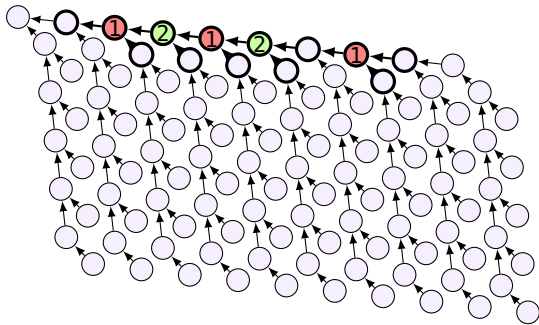
Example — $2\frac{1}{2}$ -coloring problem



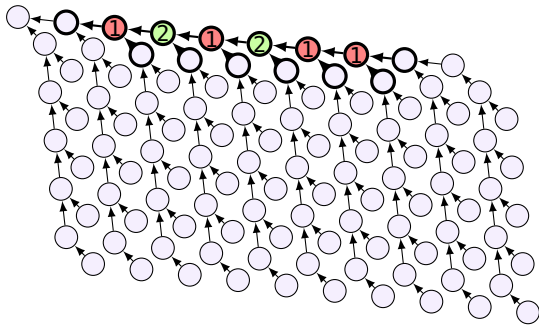
Example — $2\frac{1}{2}$ -coloring problem



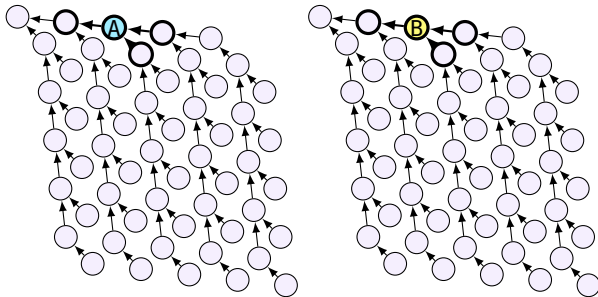
Example — $2\frac{1}{2}$ -coloring problem



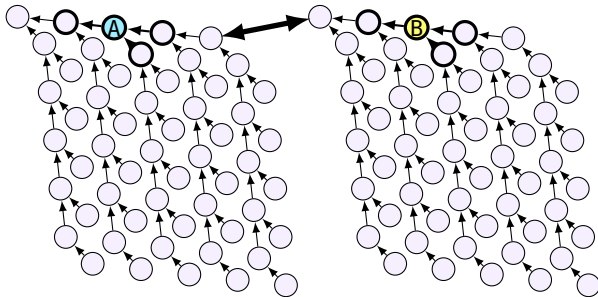
Example — $2\frac{1}{2}$ -coloring problem



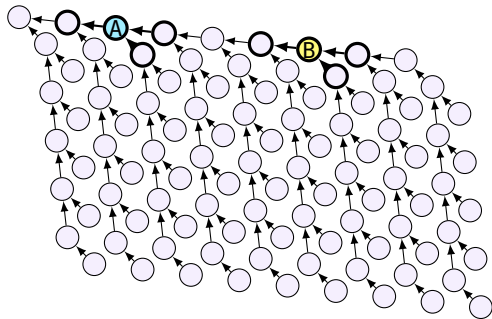
Example — $2\frac{1}{2}$ -coloring problem



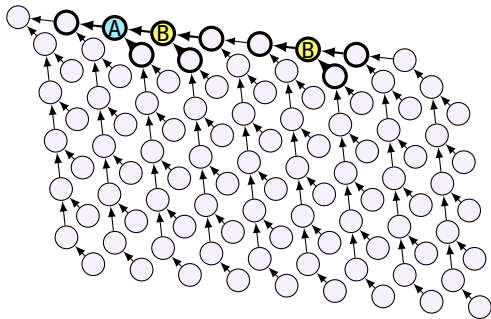
Example — $2\frac{1}{2}$ -coloring problem



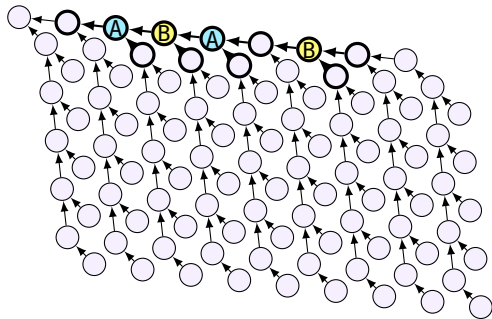
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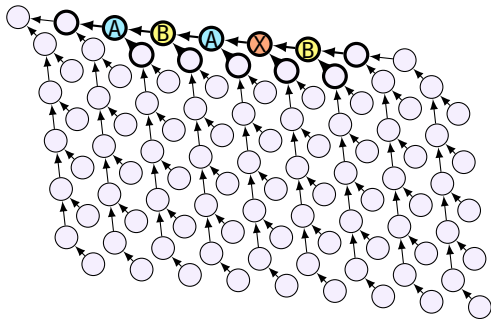
Example — $2\frac{1}{2}$ -coloring problem



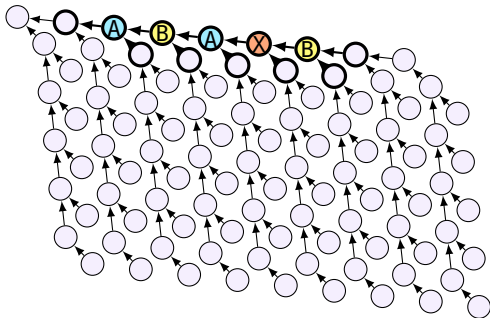
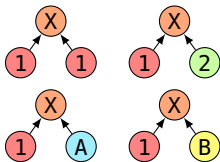
Example — $2\frac{1}{2}$ -coloring problem



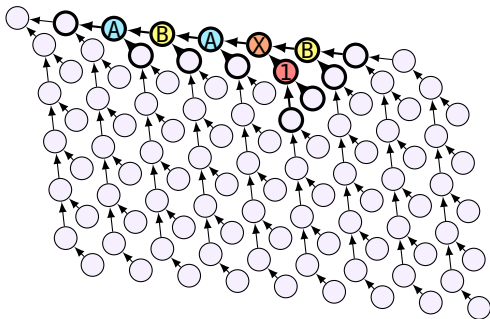
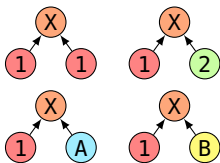
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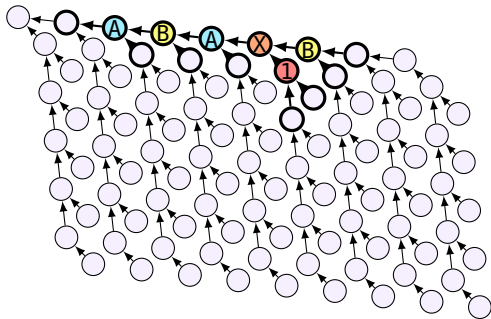
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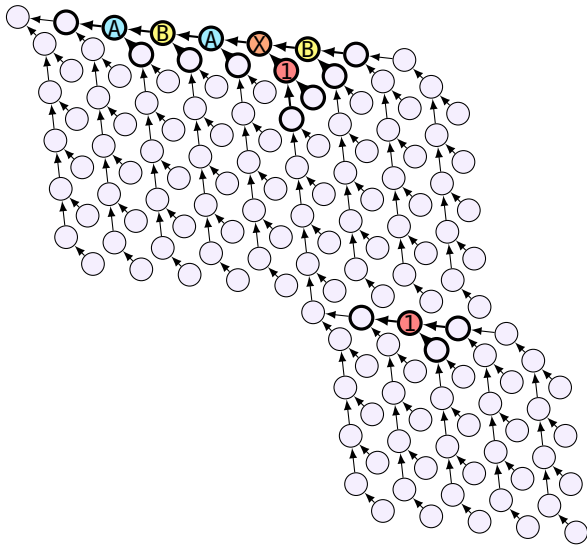
Example — $2\frac{1}{2}$ -coloring problem



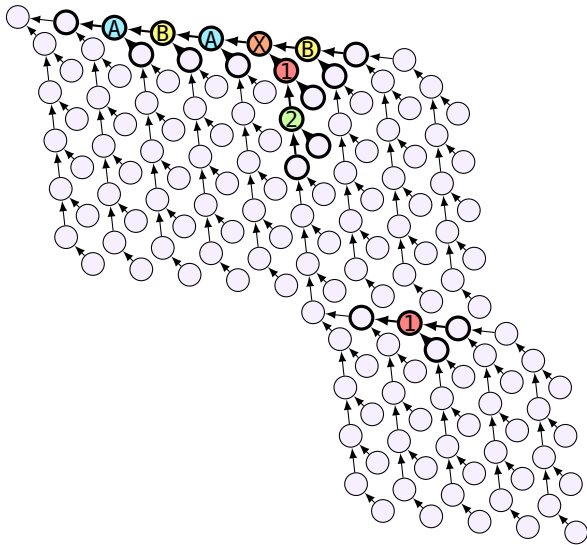
Example — $2\frac{1}{2}$ -coloring problem



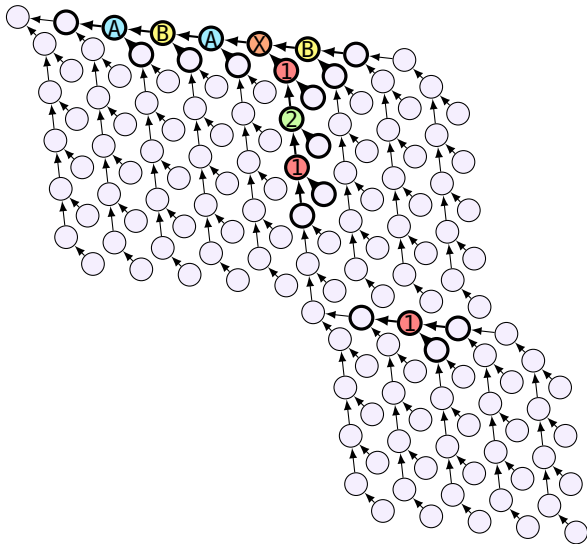
Example — $2\frac{1}{2}$ -coloring problem



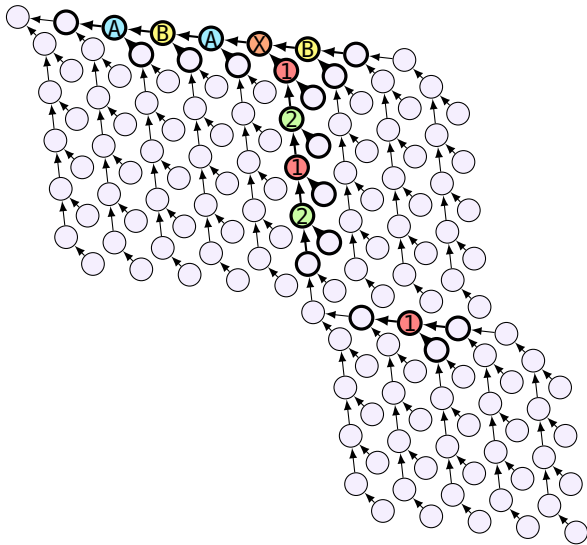
Example — $2\frac{1}{2}$ -coloring problem



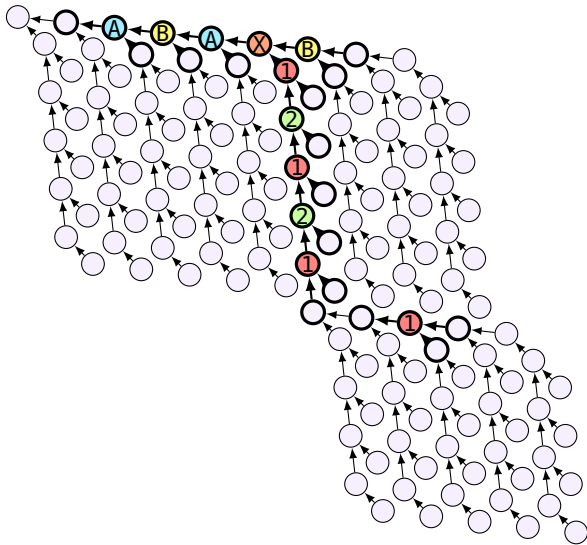
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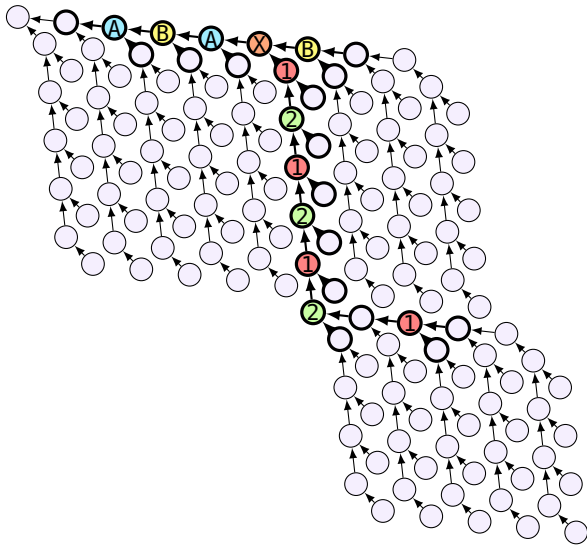
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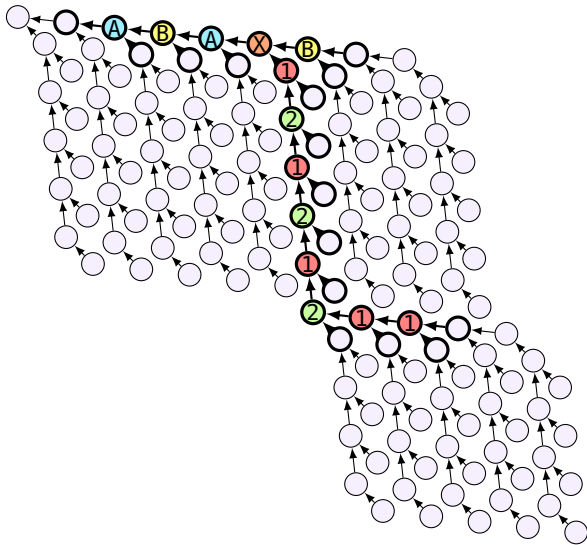
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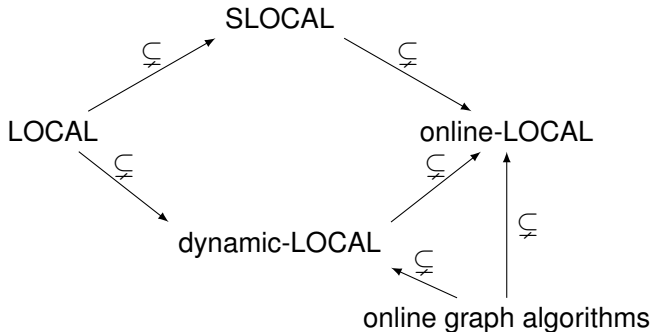
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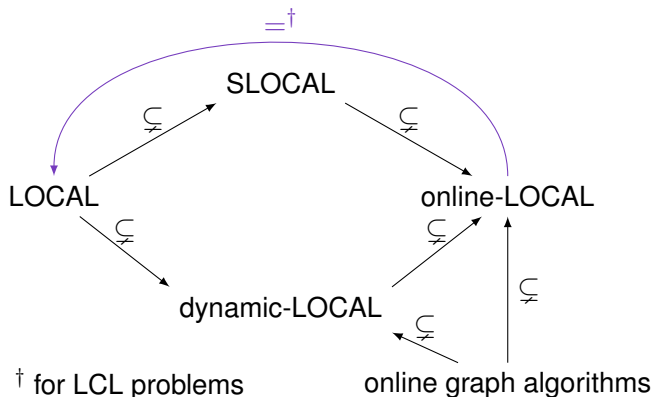
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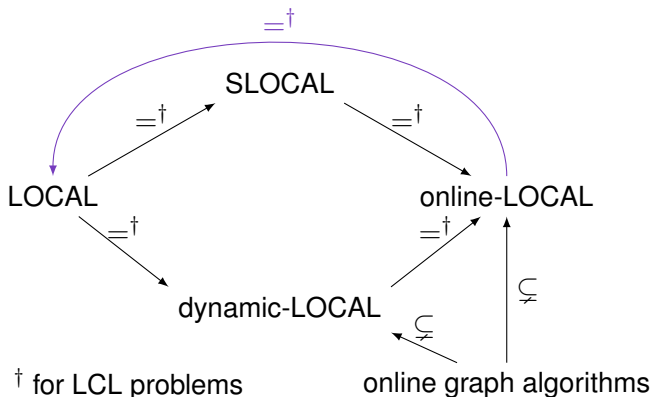
Contributions — Broader context



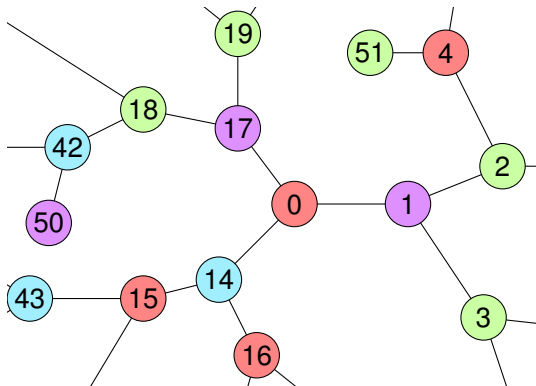
Contributions — Broader context



Contributions — Broader context



Open questions



Question

What happens with LCL problems in unrooted trees?

Question

How does randomness affect online-LOCAL?