

Locality in Online, Dynamic, Sequential, and Distributed Graph Algorithms

Amirreza Akbari¹ Navid Eslami¹³ Henrik Lievonen¹ Darya Melnyk¹²
Joonas Särkijärvi¹ Jukka Suomela¹

¹Aalto University, Finland

²TU Berlin, Germany

³Sharif University of Technology, Iran

July 14, 2023

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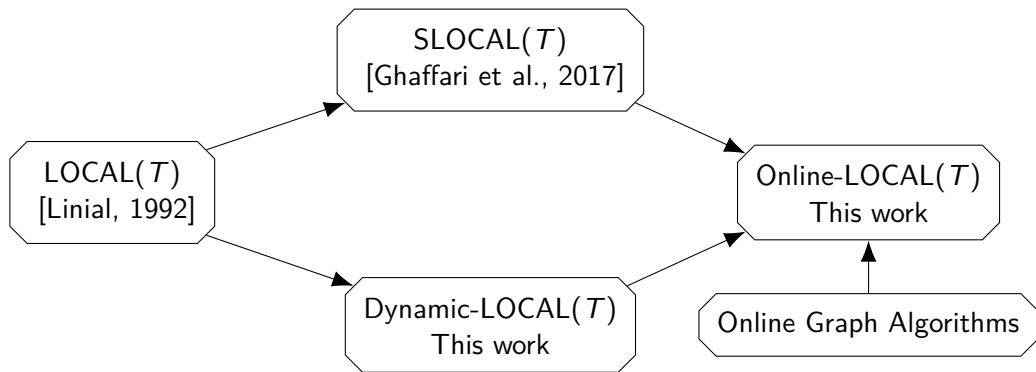
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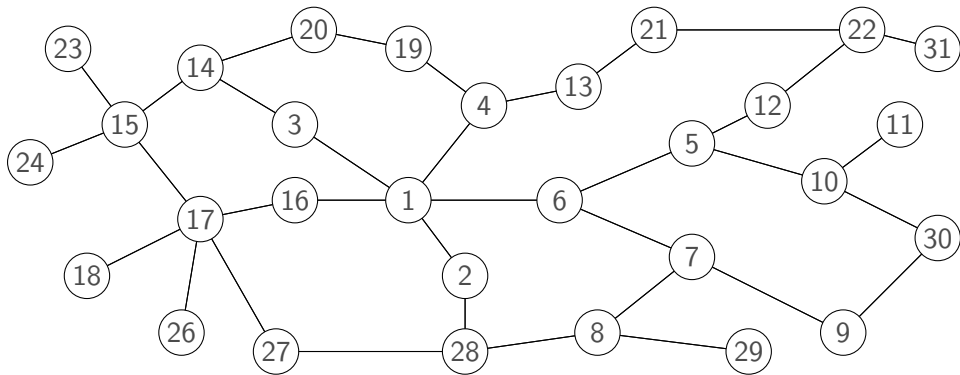
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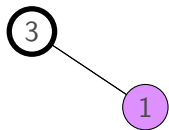
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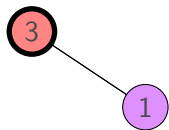


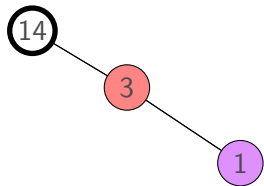


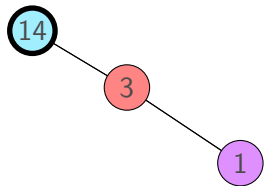
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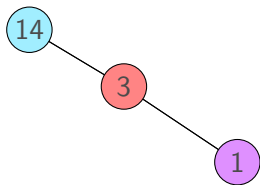


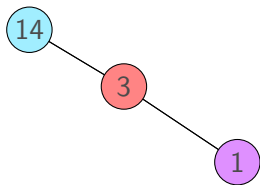


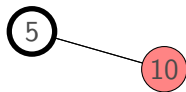
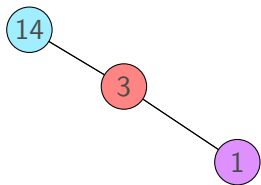


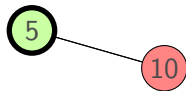
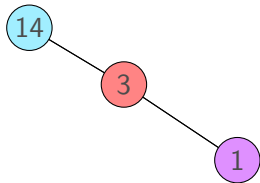


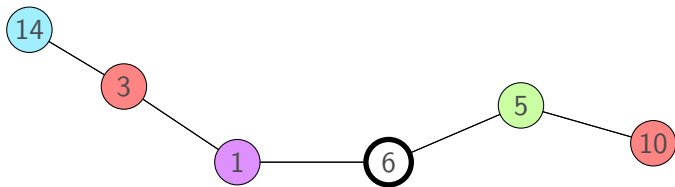


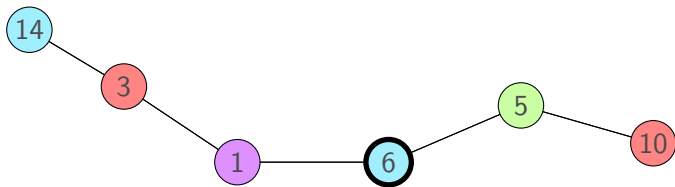


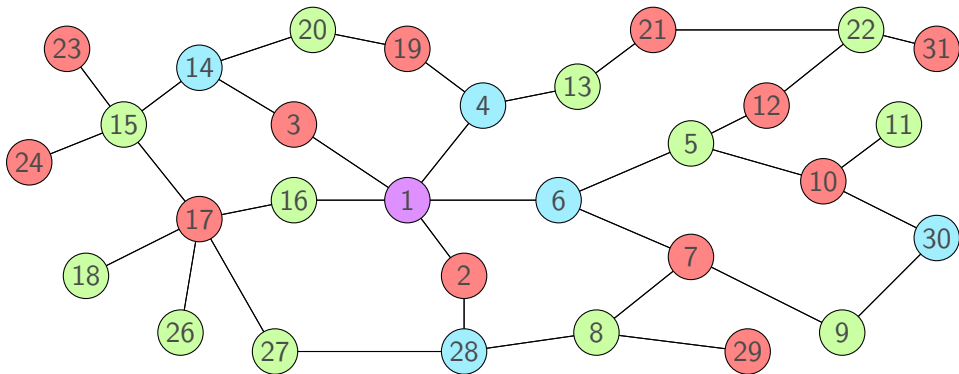






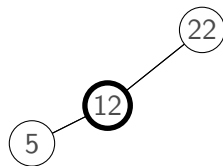




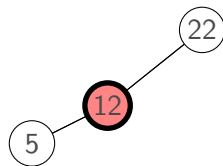


Local Advice: Online-LOCAL ($T = 1$)

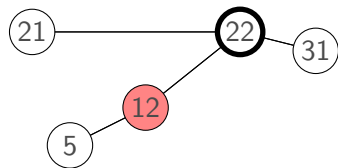
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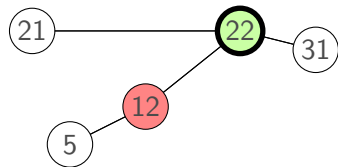
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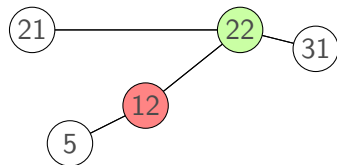
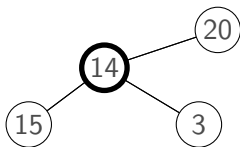
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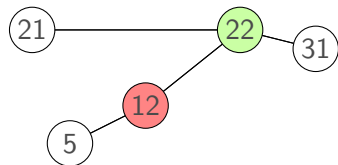
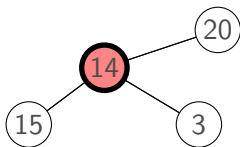
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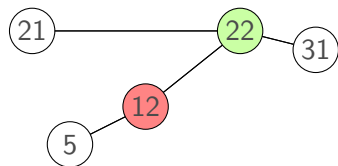
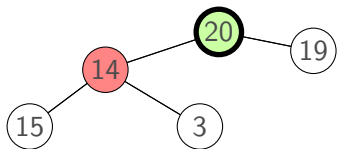
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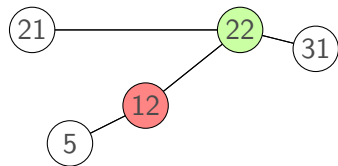
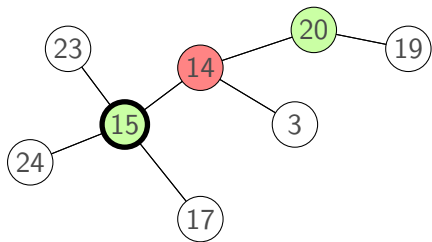
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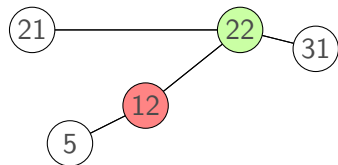
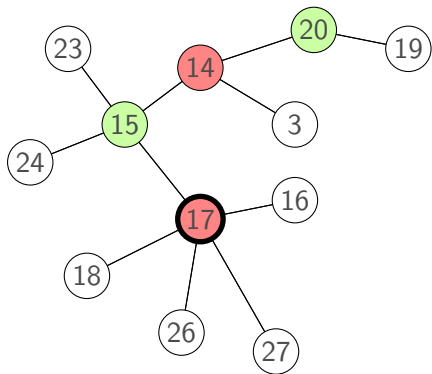
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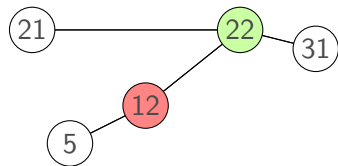
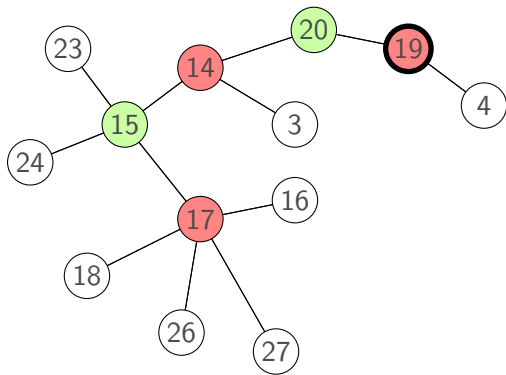
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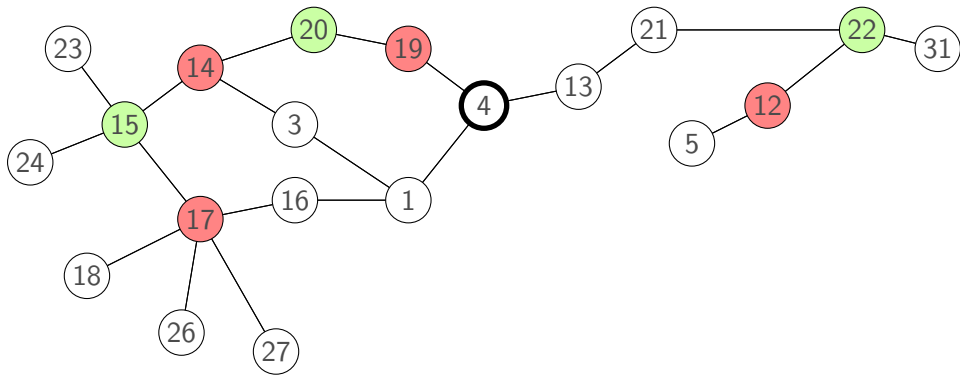
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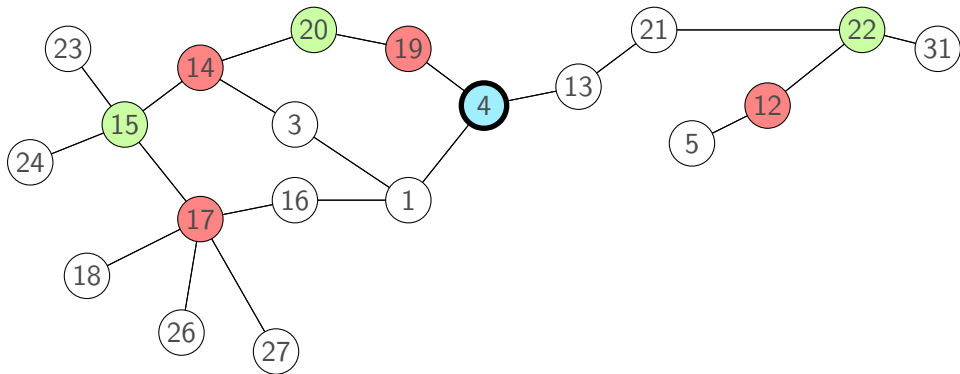
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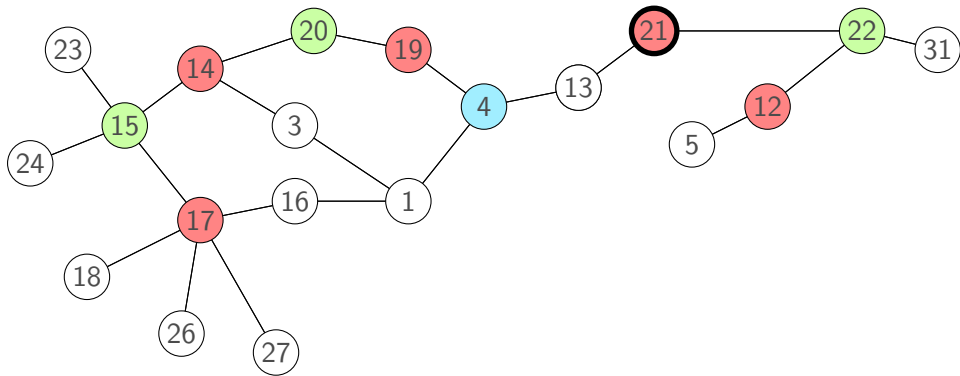
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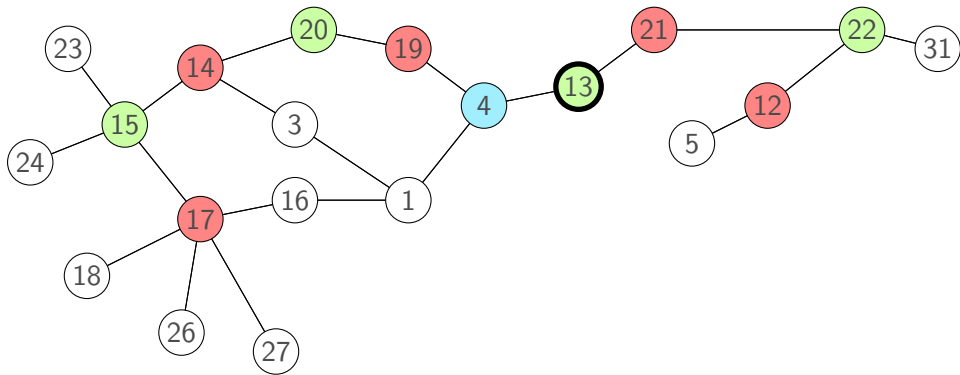
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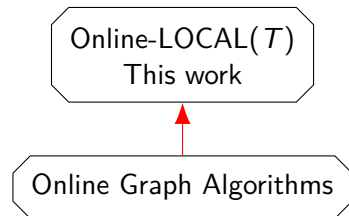
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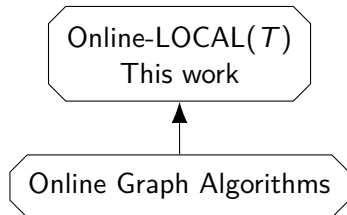
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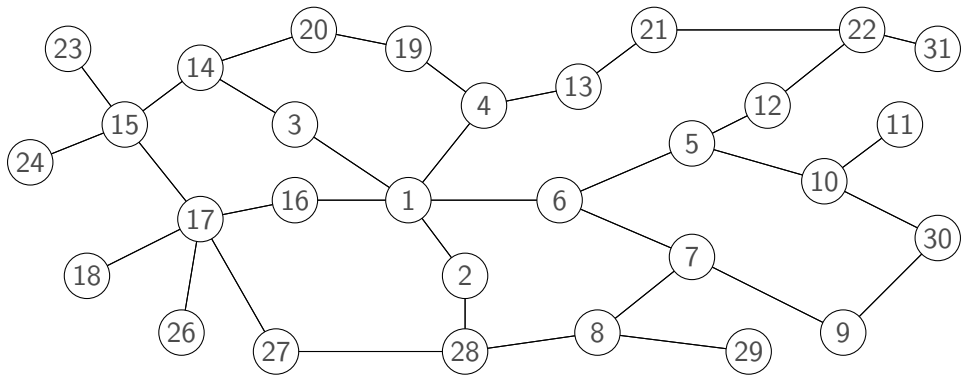
Online Graph Algorithms



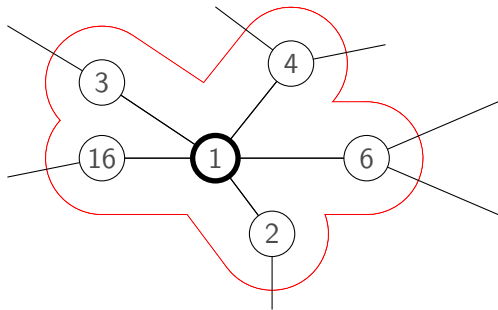
LOCAL(T)
[Linial, 1992]



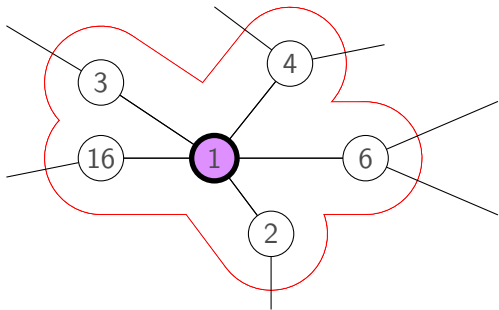
Distributed Local Algorithms: LOCAL ($T = 1$)



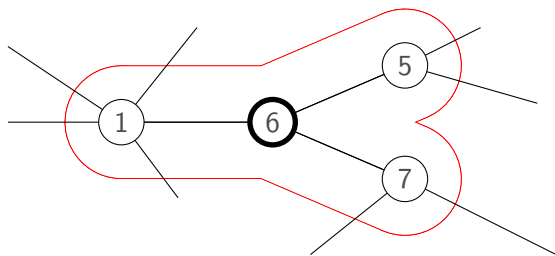
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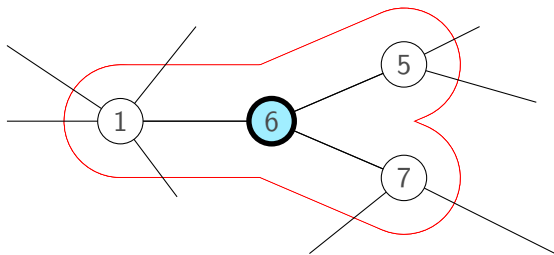
Distributed Local Algorithms: LOCAL ($T = 1$)



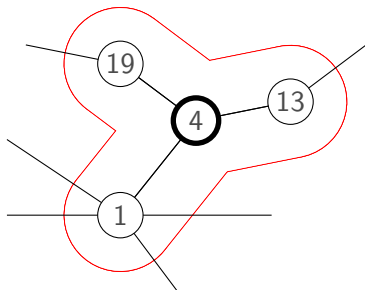
Distributed Local Algorithms: LOCAL ($T = 1$)



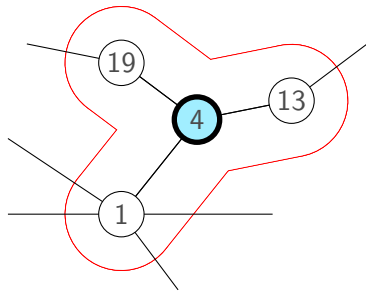
Distributed Local Algorithms: LOCAL ($T = 1$)



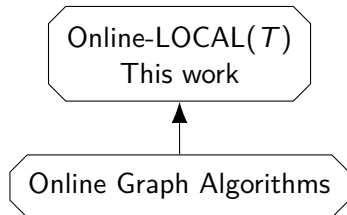
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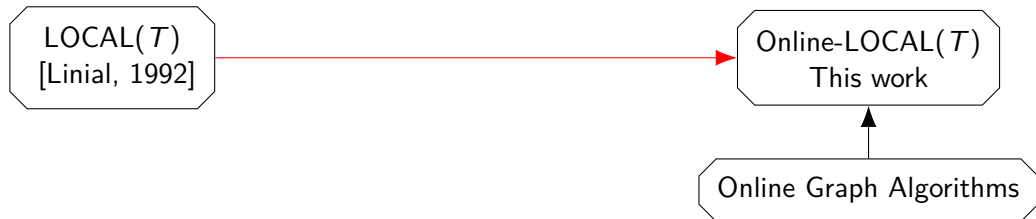


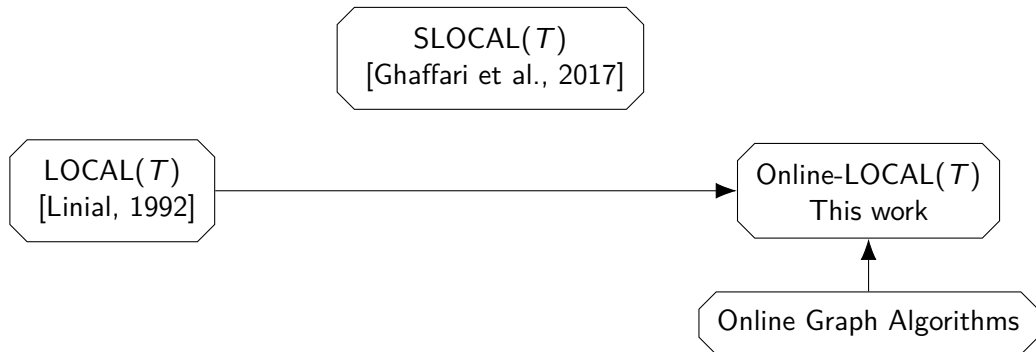
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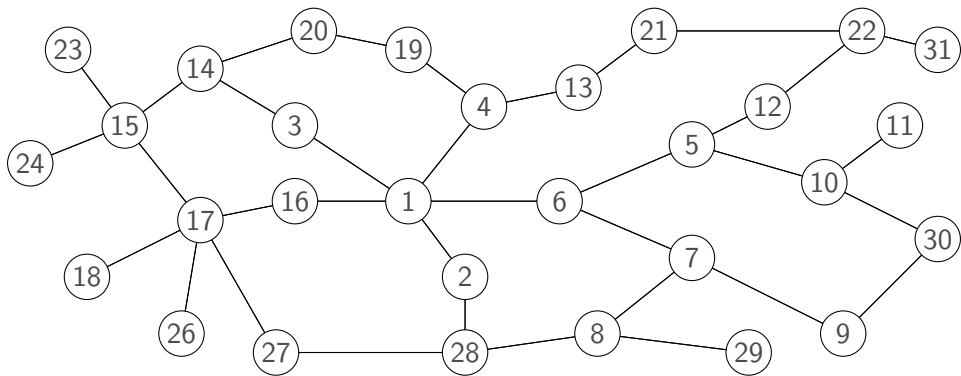
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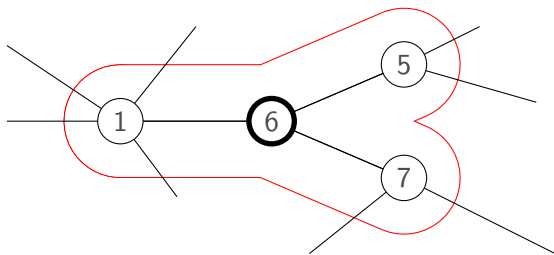




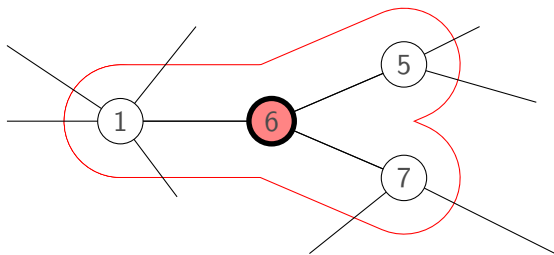
Sequential Local Algorithms: SLOCAL ($T = 1$)



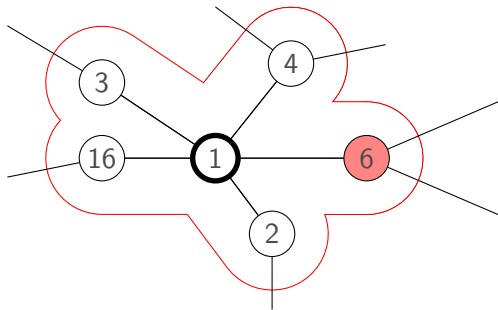
Sequential Local Algorithms: SLOCAL ($T = 1$)



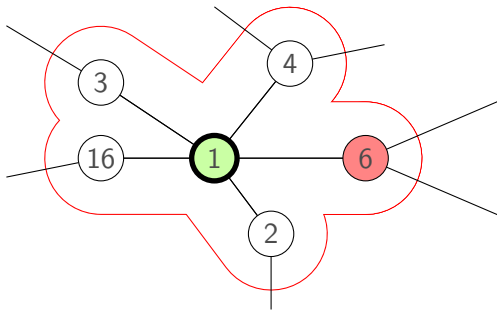
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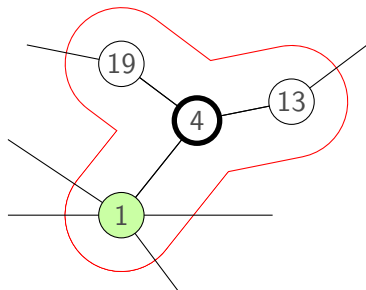
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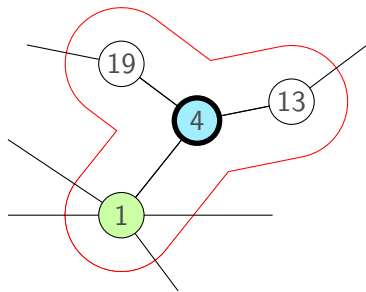
Sequential Local Algorithms: SLOCAL ($T = 1$)



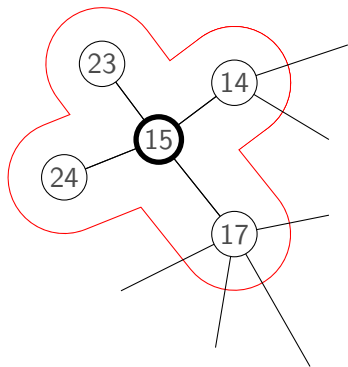
Sequential Local Algorithms: SLOCAL ($T = 1$)



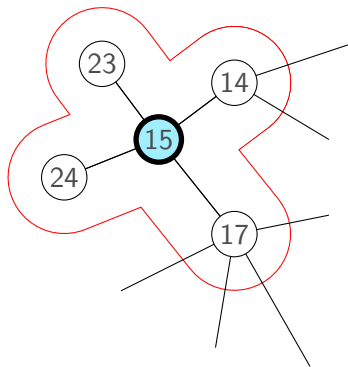
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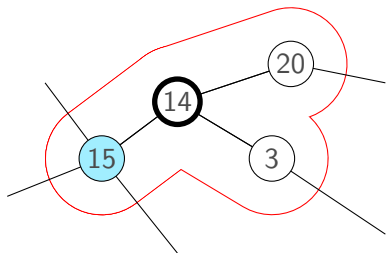
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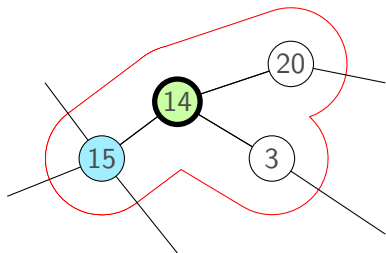
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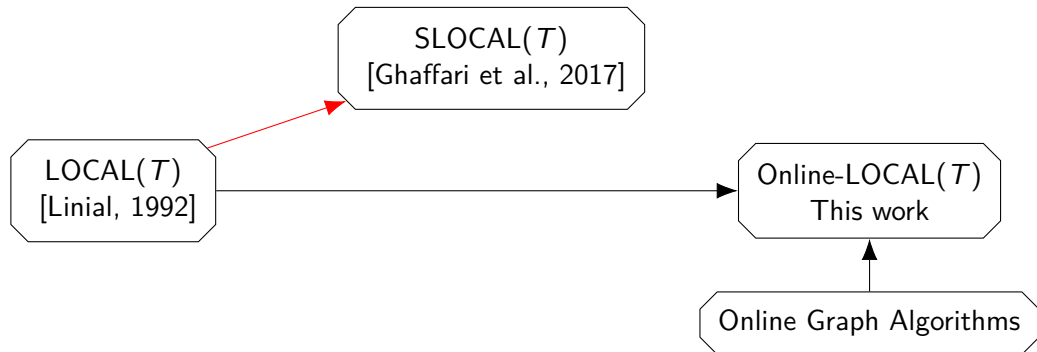
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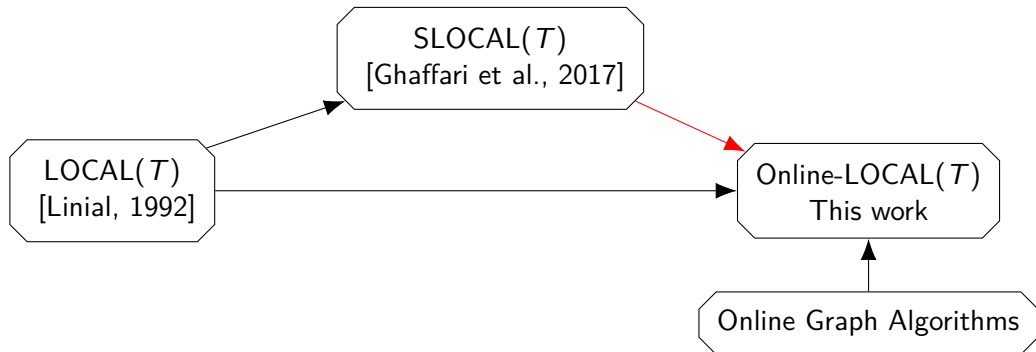


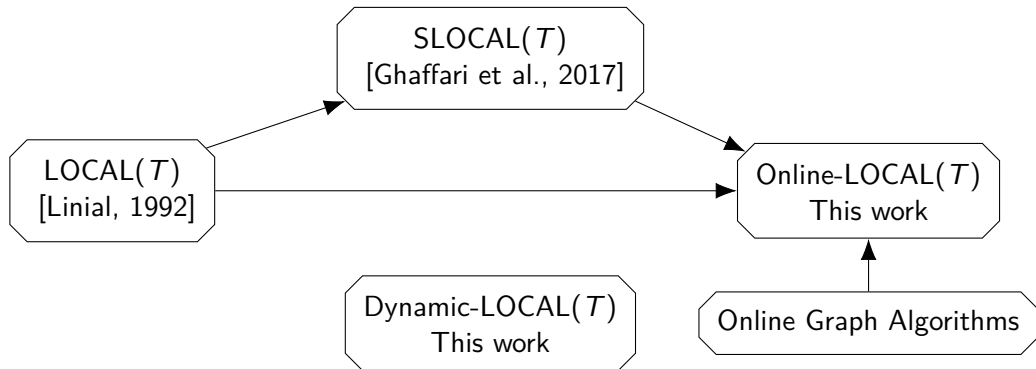
Sequential Local Algorithms: SLOCAL ($T = 1$)



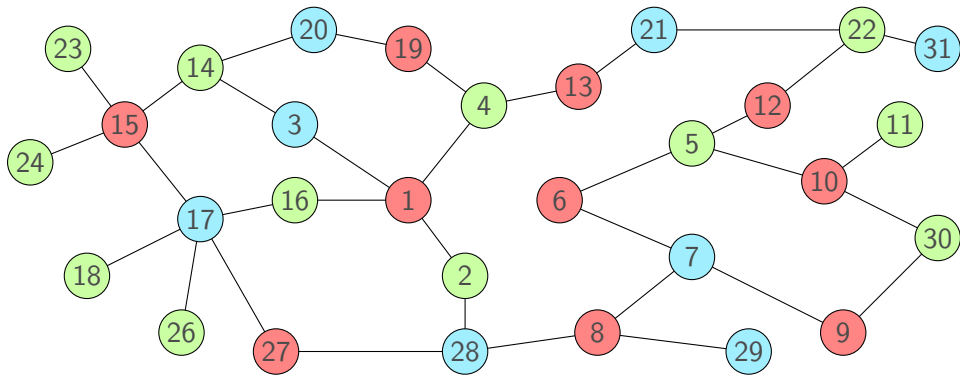




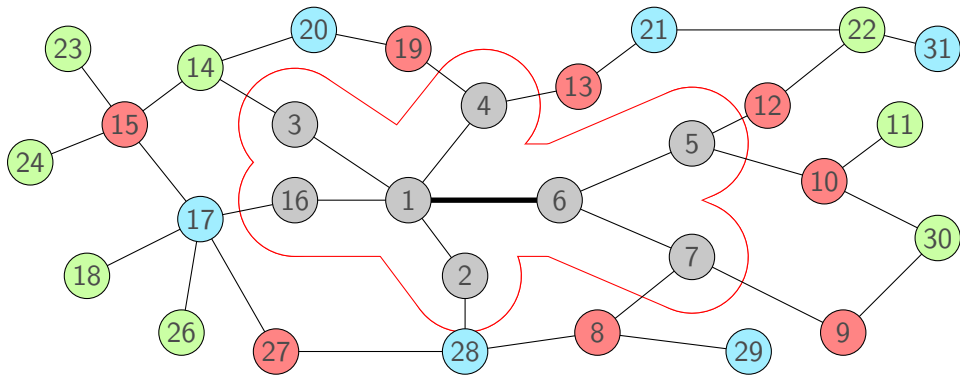




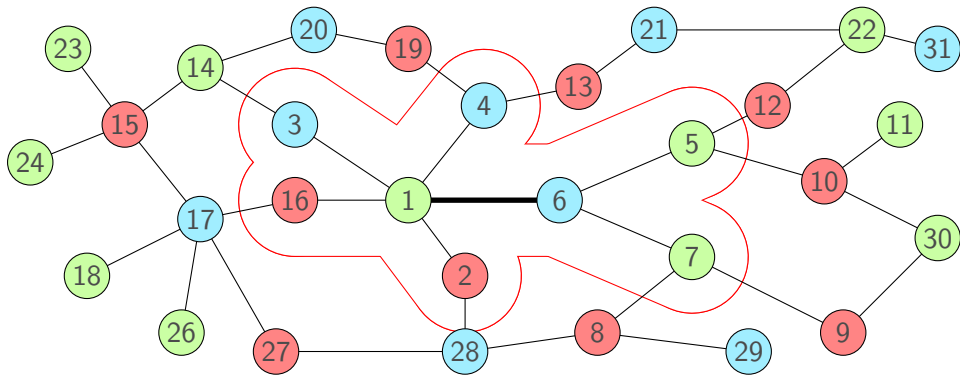
Dynamic Local Algorithms: Dynamic-LOCAL ($T = 1$)



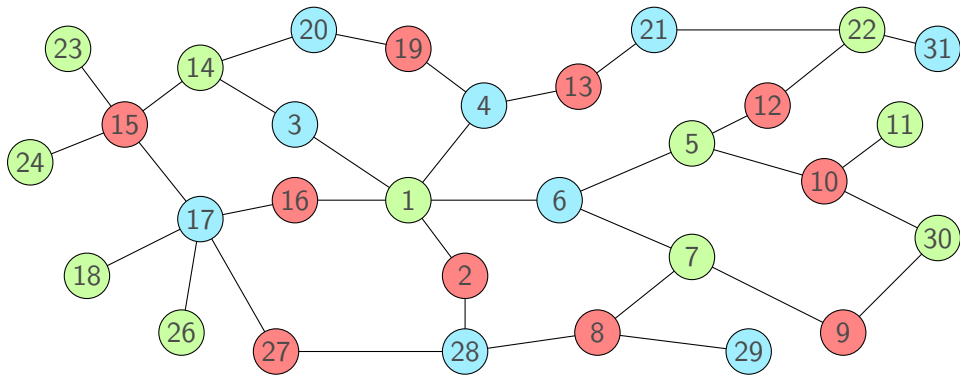
Dynamic Local Algorithms: Dynamic-LOCAL ($T = 1$)

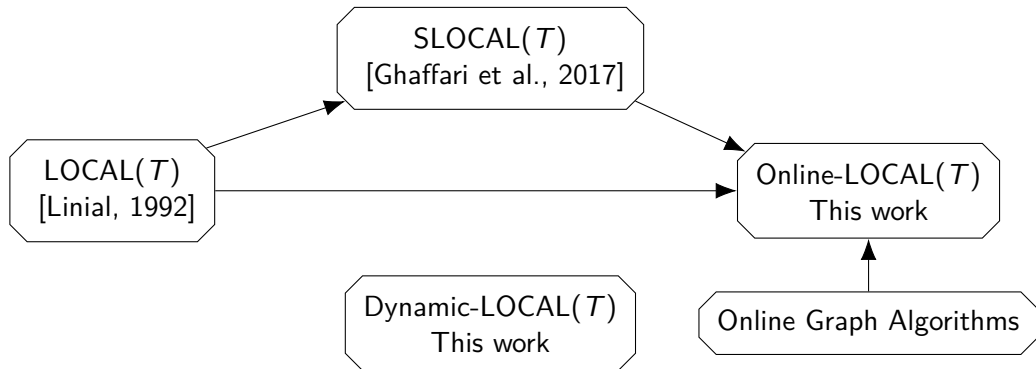


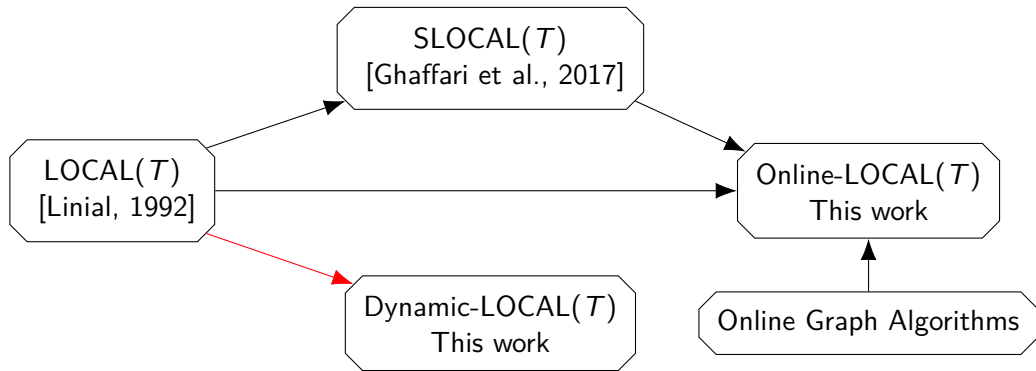
Dynamic Local Algorithms: Dynamic-LOCAL ($T = 1$)

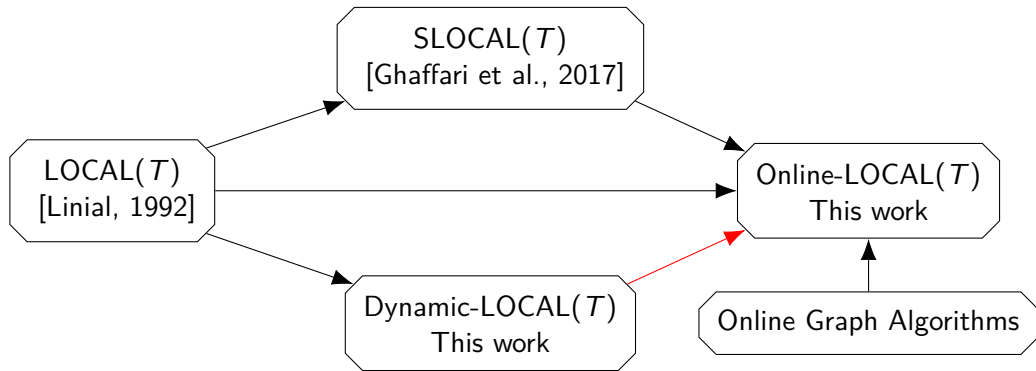


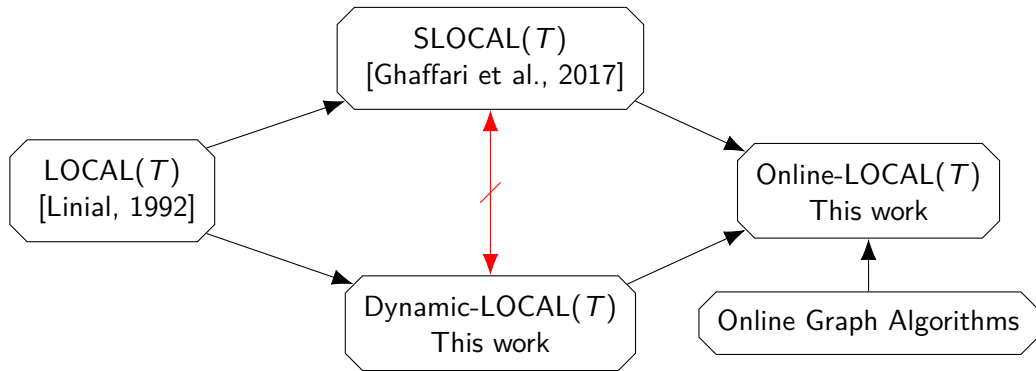
Dynamic Local Algorithms: Dynamic-LOCAL ($T = 1$)

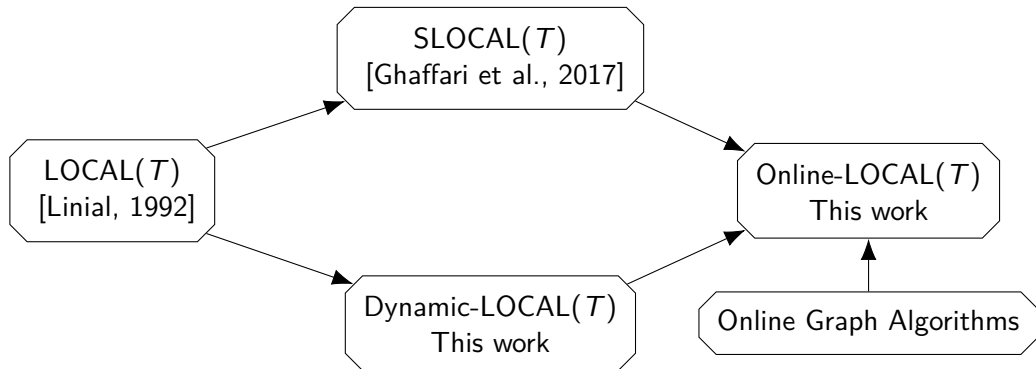


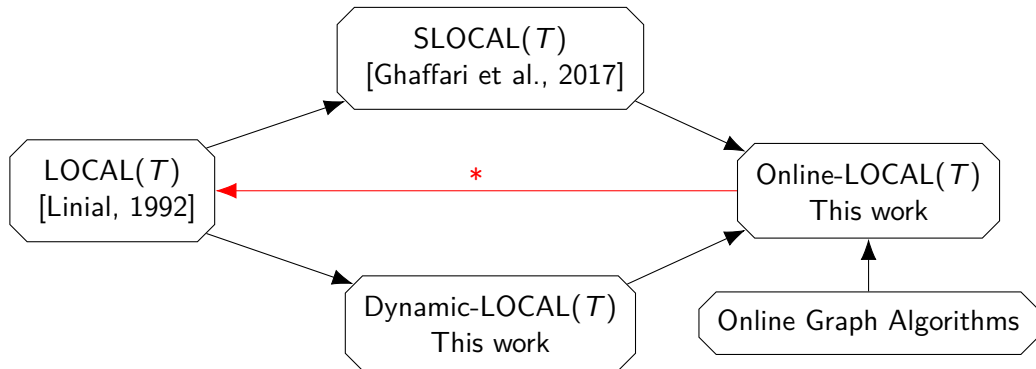




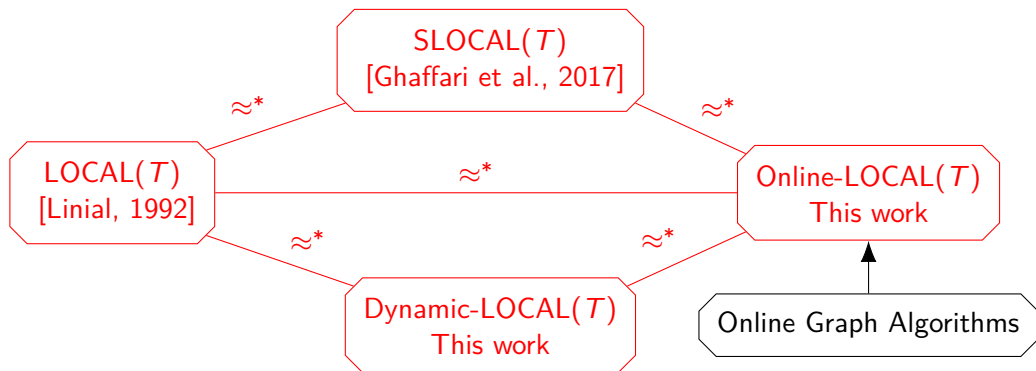






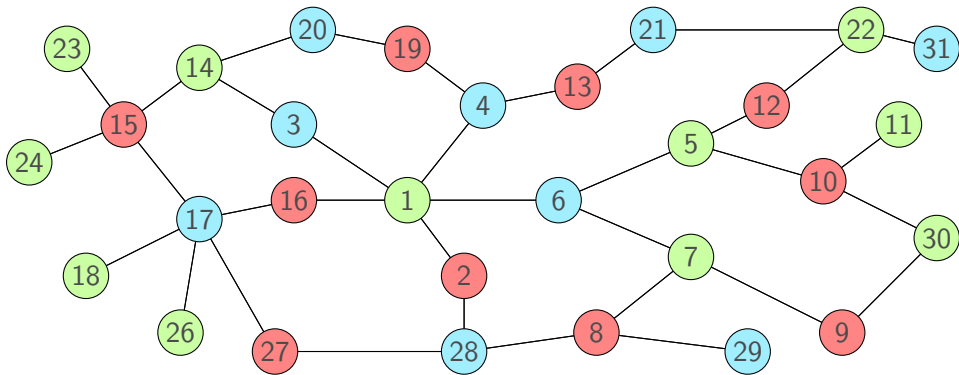


* For LCL problems in paths, cycles and rooted trees

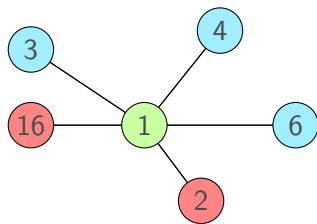


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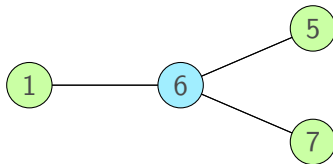
Locally Checkable Labeling (LCL) Problems



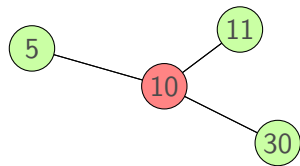
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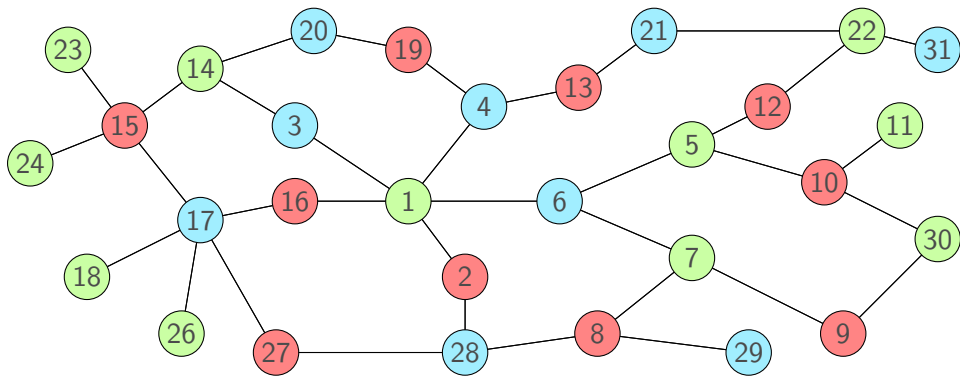
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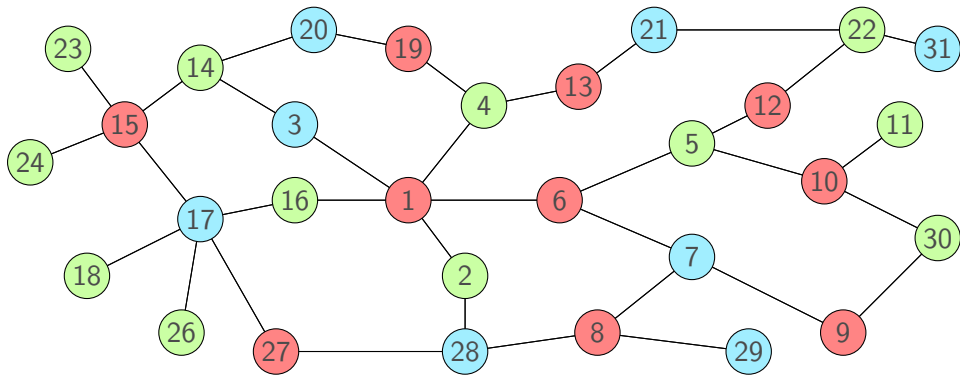
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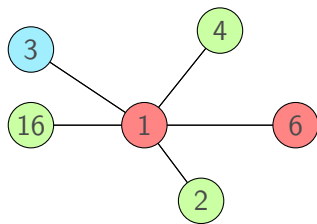
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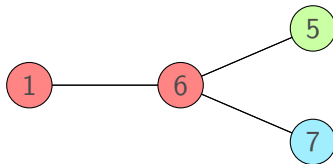
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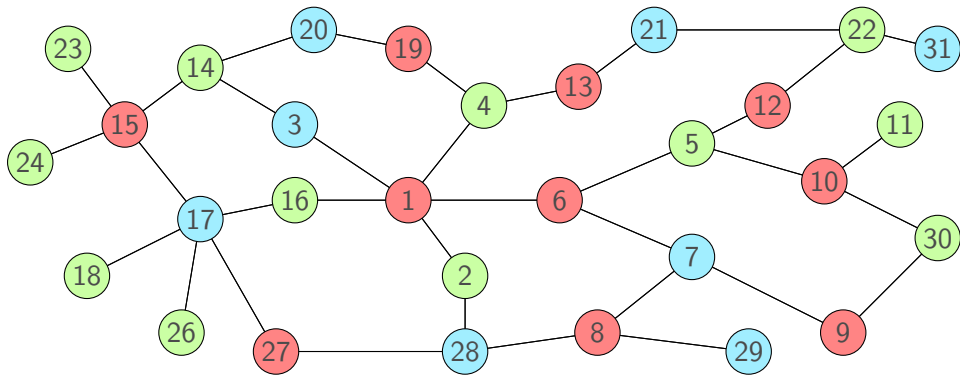
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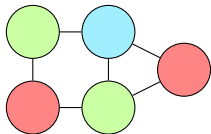
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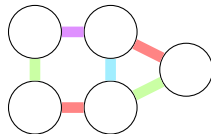
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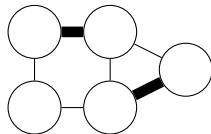
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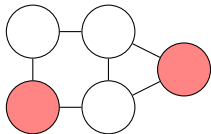
Vertex Coloring



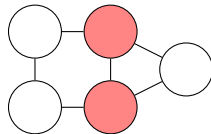
Edge Coloring



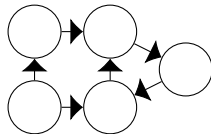
Maximal Matching



Maximal Independent Set



Minimal Dominating Set



Sinkless Orientation

Locality of Graph Coloring: Paths and Rooted Trees

colors	competitive ratio	LOCAL	SLOCAL	online-LOCAL
2	1	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
3	1.5	$\Theta(\log^* n)$	$O(1)$	$O(1)$
4	2	$\Theta(\log^* n)$	$O(1)$	$O(1)$
...				

[Linial, 1992] [Cole and Vishkin, 1986]

Locality of Graph Coloring: Paths and Rooted Trees

colors	competitive ratio	LOCAL	SLOCAL	online-LOCAL
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Locality of Graph Coloring: Grids

colors	competitive ratio	LOCAL	SLOCAL	online-LOCAL
2	1	$\Theta(n^{1/2})$	$\Theta(n^{1/2})$	$\Theta(n^{1/2})$
3	1.5			
4	2	$\Theta(\log^* n)$	$O(1)$	$O(1)$
5	2.5	$\Theta(\log^* n)$	$O(1)$	0
...				

[Brandt et al., 2017]

Locality of Graph Coloring: Grids

colors	competitive ratio	LOCAL	SLOCAL	online-LOCAL
2	1	$\Theta(n^{1/2})$	$\Theta(n^{1/2})$	$\Theta(n^{1/2})$
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Locality of Graph Coloring: Grids

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3	1.5	$\Theta(n^{1/2})$	$\Omega(n^{1/10})$	
4	2	$\Theta(\log^* n)$	$O(1)$	$O(1)$
5	2.5	$\Theta(\log^* n)$	$O(1)$	0
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[Brandt et al., 2017]

Locality of Graph Coloring: Grids

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3	1.5	$\Theta(n^{1/2})$	$\Omega(n^{1/10})$	$O(\log n)$
4	2	$\Theta(\log^* n)$	$O(1)$	$O(1)$
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...				

[Brandt et al., 2017]

Locality of Graph Coloring: Unrooted Trees

colors	competitive ratio	online graph algorithms	online-LOCAL
2	1	impossible	$\Theta(n)$
3	1.5	impossible	$O(\log n)$
...			
$\Omega(\log n)$	$\Omega(\log n)$	possible	0

Locality of Graph Coloring: Unrooted Trees

colors	competitive ratio	online graph algorithms	online-LOCAL
2	1	impossible	$\Theta(n)$
3	1.5	impossible	$O(\log n)$
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$\Omega(\log n)$	$\Omega(\log n)$	possible	0

Locality of Graph Coloring: Unrooted Trees

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$\Omega(\log n)$	$\Omega(\log n)$	possible	0

- Can we interpolate between $O(1)$ colors and $O(\log n)$ locality and $\Omega(\log n)$ colors and $O(1)$ locality in online-LOCAL?
- Can bipartite graphs be 3-colored with $o(\log n)$ locality?
- Is dynamic-LOCAL closer to LOCAL or online-LOCAL?
- How to handle randomized models?



[https://arxiv.org/
abs/2109.06593](https://arxiv.org/abs/2109.06593)